



Generalized transform technique for fluid-structural interaction during water hammer

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ABSTRACT

Unsteady flow in pipes may appear in certain circumstances and may cause undesired effects in industrial processes. The objective of this paper is to develop a new simulation model for the fluid-structure interactions (FSI) that occur in pipeline systems during water hammer. The mathematical formulation is based on water hammer equations, traditionally used in the literature, coupled with a standard beam formulation for the structure and implementing the generalized integral transform technique, GITT to treat the set of equations. The solution method is applied to a straight pipeline system subjected to axial impact load by a rapid valve closure and axial vibration of pipe, pressure and velocity of fluid are investigated. In the first step, the set of governing partial differential equations is transformed into a set of second-order differential equations in dimensionless form. In the second step the ideas in GITT approach to construct a hybrid analytical-numerical solution are implemented to obtain dynamic response of FSI during water hammer. By adopting a fourth-order eigenvalue problems and proposing eigenfunction expansions the transformed functions system results in the form of initial value problems. The explicit fourth-order Runge-Kutta and the implicit Adams-Bashforth-Moulton scheme are used to treat the set of obtained coupled ordinary differential initial value problems. Following the evaluation of time dependent variables, the analytical inversion formulas are recalled to recover the dimensionless functions for the velocity of fluid, the pressure and the axial displacement of the wall of pipe, $v(x,t)$, $p(x,t)$ and $u(x,t)$. To illustrate the capability and efficiency of the proposed scheme, Delft Hydraulic Benchmark (DHB) problem B is chosen to show the performance of the methodology. Furthermore, the convergences behavior of solution is examined for increasing truncation orders and a perfect convergence is observed.

Keywords: Fluid-structure interaction, waterhammer, Integral transforms method

1. INTRODUCTION

Since 1970's a substantial amount of research in the FSI field has been focused on understanding and quantifying interactions between the transient flow in the fluid and the resulting vibrations of the piping system. The main stream of the FSI research in fluid-filled piping systems is currently based upon the principle of coupling between the fluid and the structure at the level of a physical model represented with a set of one-dimensional partial differential equations. Skalak 1966 [1] defined a set of four linear first order partial differential equations (PDEs) for the simulations of interactions between the transient in the fluid and the axial movement of the straight section of the pipe. Skalak derived the FSI four equation models as an extension of Joukowsky's method and as the low-frequency limit of two-dimensional fluid and shell representations. He showed that this model permits solutions that are waves of arbitrary shape traveling without dispersion at the phase velocity of either liquid or pipe, but he made no attempt to solve the four equations in general. Vardy and Fan 1986 [2], Tijsseling 1996 [3], Tijsseling and Lavooij 1997 [4], Tijsseling 2003 [5], Gale and Tiselj 2000 [6], and many other researchers proved the validity and effectiveness of this model by both theoretical and experimental studies in the time and frequency domains. The linear model was so widely used, discussed and verified in practice, that it became the fundamental model in the FSI field. The models that came out from the Skalak's model are based upon essentially the same assumptions; they differ in the number of equations i.e. in the number of the tracked waves that travel along the pipe and interact with each other. These waves are axial, flexural, rotational, radial and torsional stress waves in the piping system and pressure waves in the fluid. All models have continuity and momentum balance equations for the description of the water hammer in the fluid. Elansary and Contractor 1994 [7] used Skalak's model and added gravity and friction to the water hammer part of the equations to solve the problem of the rapid valve closure in a tank-pipe-valve system. They prescribed a procedure for the optimum closure of a valve in a given time interval to minimize the reaction forces and verified it with experiments.