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Mathematical Simulation of the Heating of a Viscous Liquid Droplet Spreading over a Horizontally Oriented Cylindrical Pipe

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Abstract—Liquid film generated during steam condensation in a bundle of horizontal tubes flows down under the effect of surface forces in the form of separate droplets and streams. A mathematical model for nonisothermal spreading of a viscous liquid droplet over a horizontal surface is developed, using which it is possible to determine the temperature field and shape of such liquid droplet.

Keywords: mathematical modeling, condensation, vertical row, spreading of droplets, cylindrical surface, viscous liquid

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Nusselt's theory describes steam condensation on a vertical row of horizontal tubes, and its basic assumption is that there is continuous flow over each tube and that condensate flows over from one tube to another in the form of continuous film. However, investigations of the real mechanism through which condensate is accumulated and removed in the intertube space have shown that condensate is removed in the form of droplets or streams. These droplets and streams are generated from the mass of condensate accumulating in the layer under the tube bottom.

Experimental investigations have revealed the effects a discrete flow pattern of liquid droplets has on local heat transfer. Using the method of high-speed motion-picture recording, the authors of [1–3] have shown that the heat transfer during steam condensation on a vertical row of horizontal tubes depends in many respects on the pattern in which condensate droplets separate and fall from upper to lower tubes. In the scientific editions known to the authors of this paper, problems concerned with spreading of droplets over solid flat surfaces were studied in detail, but there is no theoretical description of nonisothermal spreading of droplets over horizontal cylindrical surfaces.

STATEMENT OF THE PROBLEM

We assume that a droplet of viscous incompressible liquid, the lateral surface of which at the initial moment of time is confined by a spherical surface and the base surface of which is confined by a cylindrical surface, begins to spread over a heated horizontal tube (Fig. 1). Heated gas flows over the droplet lateral surface. We now introduce a cylindrical system of coordinates with the origin at point O lying at the intersection of the cylindrical tube axis and the perpendicular to the tube axis passing through the center of the spot through which the droplet is contacting with the underlying surface. We also merge the Oz axis of the cylindrical system of coordinates with the cylindrical tube axis. We assume that the liquid contained in the droplet moves symmetrically with respect to the planes $z = 0$ and $\varphi = \pi/2$; therefore the droplet heating and spreading process will be studied in the region $\Omega = \{r \geq r_t, 0 \leq \varphi \leq \pi/2, z \geq 0\}$. where r_t is the tube radius, m, and $\partial\Omega$ is the moving liquid droplet boundary contacting with heated gas, m.

For finding the study region Ω at the initial moment of time, we must know the values of angle φ_0 and polar radius r for the points belonging to the droplet external boundary $\partial\Omega$.

The equation for a sphere of the droplet radius r_{dr} written in the coordinate system $Oxyz$ has the form

$$x^2 + (y - r_t)^2 + z^2 = r_{dr}^2.$$

Shifting from the Cartesian system of coordinates to the cylindrical system of coordinates we obtain

$$r^2 - 2r_t r \sin\varphi + z^2 = r_{dr}^2 - r_t^2. \quad (1)$$

The point M (see Fig. 1) has the coordinates (r_t, φ_0, 0); therefore,

$$r_t^2 - 2r_t^2 \sin\varphi_0 = r_{dr}^2 - r_t^2.$$

Hence,

$$\varphi_0 = \arcsin\left(1 - \frac{r_{dr}^2}{2r_t^2}\right).$$