

An alternative view of variational Bayes and asymptotic approximations of free energy

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Abstract Bayesian learning, widely used in many applied data-modeling problems, is often accomplished with approximation schemes because it requires intractable computation of the posterior distributions. In this study, we focus on two approximation methods, variational Bayes and local variational approximation. We show that the variational Bayes approach for statistical models with latent variables can be viewed as a special case of local variational approximation, where the log-sum-exp function is used to form the lower bound of the log-likelihood. The minimum variational free energy, the objective function of variational Bayes, is analyzed and related to the asymptotic theory of Bayesian learning. This analysis additionally implies a relationship between the generalization performance of the variational Bayes approach and the minimum variational free energy.

Keywords Variational Bayes · Local variational approximation · Variational free energy · Generalization error · Asymptotic analysis

1 Introduction

Bayesian estimation provides a powerful framework for learning from data. Recently, its asymptotic theory has been established, which supports its effectiveness for latent variable models such as the Gaussian mixture model (GMM) and hidden Markov model (HMM). More specifically, a formula for evaluating asymptotic forms of stochastic complexity or free energy was obtained and the generalization errors of statistical models have been intensively analyzed (Watanabe 2009; Yamazaki and Watanabe 2003a, 2003b, 2005; Rusakov and Geiger 2005; Aoyagi and Watanabe 2005; Yamazaki et al. 2010).

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