

# Construction and learnability of canonical Horn formulas

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**Abstract** We describe an alternative construction of an existing canonical representation for definite Horn theories, the Guigues-Duquenne basis (or GD basis), which minimizes a natural notion of implicational size. We extend the canonical representation to general Horn, by providing a reduction from definite to general Horn CNF. Using these tools, we provide a new, simpler validation of the classic Horn query learning algorithm of Angluin, Frazier, and Pitt, and we prove that this algorithm always outputs the GD basis regardless of the counterexamples it receives.

**Keywords** Query learning · Horn formulas · Canonical representation

## 1 Introduction

The research area of Query Learning studies problems of learning target concepts, often modeled as Boolean functions, through learning algorithms that interact with the target concept through a variety of protocols. Positive results consist usually in algorithms that learn a given concept class, via a given protocol, within certain query or time resources.

The most successful protocol so far is via Membership and Equivalence queries. Three major algorithms for this model are the  $L^*$  algorithm for learning regular sets in terms of deterministic finite automata (Angluin 1987); the algorithm that learns monotone DNF (closely related to its corresponding PAC version by Valiant 1984); and a sophisticated evolution of

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