

## Ranking Efficient Decision Making Units in Data Envelopment Analysis based on Reference Set and coefficient of variation

Parisa Firoozishahmirzadi(\*)

Department of Mathematics, University of Mazandaran  
Mazandaran, Babolsar, Iran  
Parisa.Firoozi@gmail.com

Kamran Fathi Far

Department of Management, Islamic Azad University of  
Tehran, Science and Research branch, Tehran, Iran  
Fathifarkamran1976@gmail.com

**Abstract.** One of the drawbacks of Data Envelopment Analysis (DEA) is the problem of lack of discrimination among efficient Decision Making Units (DMUs). A method for removing this difficulty is called changing reference set proposed by Jahanshahloo and et.al (2007). The method has some drawbacks. In this paper a method to remove this difficulty is suggested. Numerical example for illustration are given.

### I. INTRODUCTION

Data Envelopment Analysis (DEA) is a fractional programming technique that was developed by Charnes, Cooper and Rhodes (1978). It is used to measure the productive efficiency of decision making units (DMUs) and evaluate their relative efficiencies. This analysis determines the productivities of DMUs, specified as the ratio of the weighted sum of outputs to the weighted sum of inputs, comparing them to each other and determining the most efficient DMUs.

Ranking efficient DMUs is one of the problem which attracted researcher since 1978. Different Methods for this propose have been suggested, see paper by Adler and et.al (2002). Most of the these methods are not allow to rank non extreme efficient points. The method suggested by Sexton called cross efficiency rank all kind of efficient DMUs. The modified methods of Sexton method is one of the most used for this propose. Changing reference set for ranking efficient DMUs was suggested by Jahanshahloo and et.al 2007 the main drawbacks in this Method are as follows:

- 1) In cas of having no inefficient DMUs the method can not be used.
- 2) The main idea for ranking efficient is average deviation of inefficient DMUs from the original score, which seems to be not fair.

The first difficulty is not discussed in the paper, but we propose a method for removing second problem.

The rest of the paper contains the following by subject: In section 2 we explain DEA methods for measuring efficiency of DMUs, in

section 3 Jahanshahloo and et.al (2007) method will be discussed. Section 4 contains the proposed method. Numerical example for comparing the methods and illustration are given in Section 5. The last section summaries and concludes.

### II. DATA ENVELOPMENT ANALYSIS (DEA)

#### BACKGROUND

The most basic Data Envelopment Analysis (DEA) model is the CRS (Constant Return to Scale) which was proposed by Charnes et al. in 1978. The basic idea of the CRS model is the following: The efficiency of an observed DMU (Decision Making Unit) which is the organization to be evaluated, can be measured by the ratio output per input, i.e., how well DMU can convert its inputs into its outputs. As we usually work in situations where we face multiples inputs and outputs, we are going to form a unique virtual output and a unique virtual input, for the observed  $DMU_p$ , by the yet unknown weights  $v_i$  and  $u_r$ . By using Linear Programming (LP), we can find the weights that maximize the ratio output per input through the model:

$$\begin{aligned}
 & \text{Max} && \sum_{r=1}^s u_r y_{rp} \\
 & \text{S.t} && \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} \leq 0 \quad j = 1, 2, \dots, n \\
 & && \sum_{i=1}^m v_i x_{ip} = 1 \\
 & && u_r \geq \epsilon \quad r = 1, 2, \dots, s \\
 & && v_i \geq \epsilon \quad i = 1, 2, \dots, m
 \end{aligned} \tag{1}$$

Where  $x_{ij}$  is the data of input  $i$  on the  $DMU_j$ ,  $y_{rj}$  is the data of the output  $r$  on the  $DMU_j$ ,  $v_i$  is the weight of the input  $i$  and  $u_r$  is the weight of the output  $r$ . The dual form of Eq. (1) is: