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A Theory of Minimal 3D Point to 3D Plane Registration and Its Generalization

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Abstract Registration of 3D data is a key problem in many applications in computer vision, computer graphics and robotics. This paper provides a family of minimal solutions for the 3D-to-3D registration problem in which the 3D data are represented as points and planes. Such scenarios occur frequently when a 3D sensor provides 3D points and our goal is to register them to a 3D object represented by a set of planes. In order to compute the 6 degrees-of-freedom transformation between the sensor and the object, we need at least six points on three or more planes. We systematically investigate and develop pose estimation algorithms for several configurations, including all minimal configurations, that arise from the distribution of points on planes. We also identify the degenerate configurations in such registrations. The underlying algebraic equations used in many registration problems are the same and we show that many 2D-to-3D and 3D-to-3D pose estimation/registration algorithms involving points, lines, and planes can be mapped to the proposed framework. We validate our theory in simulations as well as in three real-world applications: registration of a robotic arm with an object using a contact sensor, registration of planar city models with 3D point clouds obtained using multi-view reconstruction, and registration between depth maps generated by a Kinect sensor.

Keywords 3D-to-3D registration · Pose estimation · Point-to-plane registration · Minimal solution · Correspondence problem

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1 Introduction and Previous Work

Pose estimation refers to the estimation of 6-degree-of-freedom (6-DoF) object pose using sensor measurements (e.g., images, 3D point clouds) and prior knowledge (e.g., a 3D model) of the object. This is achieved by registering the 3D data from the sensor to the known 3D model of the object. Such registration algorithms play a major role in numerous applications including object recognition, tracking, localization and mapping, augmented reality, and medical image alignment. Recent progress in the availability of 3D sensors such as Kinect (Shotton et al. 2011) at reasonable cost has further accelerated the need for such problems. The registration problem can generally be seen as two subproblems: a correspondence problem and a problem of pose estimation given the correspondence. Both of these problems are intertwined, and the solution of one depends on the other. This paper addresses the solution to both problems, although the major emphasis is on the second one.

Several 3D-to-3D registration scenarios are possible depending on the representation of the two 3D datasets: 3D points to 3D points, 3D lines to 3D planes, 3D points to 3D planes, etc. (Olsson et al. 2006). Iterative closest point (ICP) and its variants have been the gold standard in the last two decades (Besl and McKay 1992; Fitzgibbon 2003). These algorithms perform very well with a good initialization. Hence the main unsolved problem is the initial coarse registration. The registration of 3D lines to 3D planes and the registration of 3D points *with normals* to 3D planes were considered in Chen (1991), Grimson and Lozano-Pérez (1983). In contrast to their work, we register 3D points without normals to 3D planes. Note that the presence of normals makes the problem much simpler compared to the case without the normals. Recently, there have been several registration algorithms that focus on solving both the correspondence