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A Survey and Comparison of Discrete and Continuous Multi-label Optimization Approaches for the Potts Model

Claudia Nieuwenhuis · Eno Töppe · Daniel Cremers

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Abstract We present a survey and a comparison of a variety of algorithms that have been proposed over the years to minimize multi-label optimization problems based on the Potts model. Discrete approaches based on Markov Random Fields as well as continuous optimization approaches based on partial differential equations can be applied to the task. In contrast to the case of binary labeling, the multi-label problem is known to be NP hard and thus one can only expect near-optimal solutions. In this paper, we carry out a theoretical comparison and an experimental analysis of existing approaches with respect to accuracy, optimality and runtime, aimed at bringing out the advantages and short-comings of the respective algorithms. Systematic quantitative comparison is done on the Graz interactive image segmentation benchmark. This paper thereby generalizes a previous experimental comparison (Klodt et al. 2008) from the binary to the multi-label case.

Keywords Multi-label · Survey · Comparison · Optimization · Markov random fields · Partial differential equations

1 Introduction

The optimization of energies with respect to a set of variables which can take one of multiple labels is among the central algorithmic challenges in computer vision and image analysis. The prototypical example of multi-label problems is multiregion image segmentation, where every pixel is assigned one of finitely many region labels. Apart from segmentation, continuous estimation problems such as denoising, deblurring, stereo and optical flow can be approximated as a multi-label problem on a discretized label space. We will restrict our attention to algorithms which aim at minimizing a specific class of multiregion segmentation functionals, often referred to as the *Potts problem* in the MRF community or the *minimal partition problem* in the PDE community.

Over the years, numerous algorithms have been proposed to tackle multi-label optimization problems, both in the community of *partial differential equations (PDEs)* and in the community of *Markov random fields (MRFs)*. As a result one may ask how these algorithms compare in theory and in practice. Klodt et al. (2008) presented an experimental comparison of discrete and continuous optimization approaches for the specific case of *binary* labeling problems. In recent years, the focus has shifted from binary labeling to the more general multi-label problem, with a multitude of competing algorithms to solve it. The contribution of this paper is to provide a systematic theoretical and experimental comparison of algorithms for multi-label problems of the minimal partition type, pointing out relations, equivalences and differences.

In general, the segmentation task can be formulated as an energy minimization problem. In the spatially discrete setting, this energy is defined on a set of nodes, leading to MRF problems whose solution is often calculated using graph cut methods. In the spatially continuous setting, the respective optimality conditions for the continuous energy are written in terms of a set of partial differential equations which are then solved on a discrete grid.

The contributions of this paper are twofold: Firstly, we present relations between the following relaxations of multi-label optimization problems from the MRF and PDE communities:

C. Nieuwenhuis (✉) · E. Töppe · D. Cremers
Departments of Computer Science and Mathematics,
TU München, Germany
e-mail: claudia.nieuwenhuis@in.tum.de