One Dimensional Optimal System and Exact Solutions of the Fitness Equations

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Abstract

In this paper, by applying the Lie symmetry method we will obtain the lie symmetry group of the Fitness equation. Also the one dimensional optimal system of Lie algebra is obtained. Using the optimal system, similarity reductions of this equation will be constructed and some group invariant solutions are obtained.

Keywords: Lie symmetries, Lie symmetry group, Infinitesimal generators, Optimal System.

1. INTRODUCTION

Mathematical modeling is a basis for analyzing physical phenomena. Almost all fundamental equations of mathematical physics are nonlinear, and in general, are very difficult to solve explicitly. Group analysis is a method for constructing exact solutions of differential equations. This method uses the symmetry properties for constructing exact solutions**.**
There are many real-world situations that deal with quadratics and parabolas. Throwing a ball, shooting a canon, diving from a platform and hitting a golf ball are all examples of situations that can be modeled by quadratic functions. The purpose of this paper is to use Lie group analysis to obtain some exact solutions.
Let us introduce a more general class of quadratic operators in the fitness equation that is given by:

$$u_t + uu_{xxxx} + \beta u_x u_{xxx} + \gamma\left(u_{xx}\right)^2 = 0. \qquad (1)$$

where u := u(x, t) is a real function for all x, t ∈ R and γ, $\beta \geq 0$. Lie group analysis is a very powerful tool for studying general properties of differential equations and for finding their solutions. A number of papers, including [2,3,5,6,7,8,9,11], have been devoted the application of Lie group analysis to equations in financial mathematics.
We shall now present Lie group method for Equation (1). A Lie point symmetry of a partial differential equation (PDE) is an invertible transformation of the dependent and independent variables that leaves the equation unchanged. In general, determining all the symmetries of a partial differential equation is a formidable task [9]. Once one has determined the symmetry group of a system of differential equation, a number of applications become available. To start with, one can directly use the defining property of such a group and construct a new solution to the system from known ones.

2. Lie symmetry analysis

In this section, we will provide the background definitions and results. A system of n-th order differential equations in p independent and q dependent variables is given as a system of equations