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Extended Taylor series and interpolation of physically meaningful functions

Slawomir Sujecki

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Abstract An extension of the concept of the Taylor series to arbitrary functions that are physically meaningful is presented. The series is obtained using a matrix to describe the behaviour of the function at any position of discontinuity while performing the Taylor series expansion in the rest of the considered domain. The matrix is derived from the continuity conditions that are inherent to a particular physical problem. This allows an introduction of function classes that are relevant from the physical problem point of view. Several illustrative examples of physically meaningful function classes are derived. It is then demonstrated that using the derived function classes and the concept of the extended Taylor series one can obtain the interpolation formulae fitted for a particular class of physically meaningful functions. The application of the interpolation formulae based on these novel concepts shows clear advantages over the standard approach. Finally, it is also shown that the developed concept of the extended Taylor series and classes of physically meaningful functions can be used for the extrapolation and an elegant derivation of finite difference approximations for physical problems.

Keywords Taylor series · Finite differences · Interpolation · Photonic device modelling

1 Introduction

Design and modelling of optical and optoelectronic devices involves handling functions that are either discontinuous or have discontinuous derivatives. For example, all the derivatives of the transverse magnetic field component for a TM mode of an optical slab waveguide are discontinuous; likewise the temperature distribution in a medium with a stepwise discontinuous distribution of the heat conductivity has a discontinuity in the distribution of the first derivative of the temperature. Also post processing of results that were calculated by

S. Sujecki (✉)
The Department of Electrical and Electronic Engineering, The University of Nottingham, Nottingham NG7 2RD, UK
e-mail: slawomir.sujecki@nottingham.ac.uk