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ORIGINAL RESEARCH PAPER

Non-linear diffusion of image noise with minimal iterativity

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Abstract Non-linear diffusion (ND) is an iterative difference equation used in several image processing applications such as denoising, segmentation, or compression. The number of iterations required to achieve optimal processing can be very high, making ND not suitable for real-time requirements. In this paper, we study how to reduce complexity of ND so as to achieve minimal number of iterations for real-time image denoising. To do this, we first study the relations between parameters of the iterative equation: the number of iterations, the time step, and the edge strength. We then proceed by estimating the minimally required number of iterations to achieve effective denoising. Then, we relate the edge strength to the number of iterations, to noise, and to the image structure. The resulted minimal iterativity ND is very fast, while still achieves similar or better noise reduction compared to related ND work. This paper also shows how the proposed spatial filter is suitable for structure-sensitive object segmentation and temporal noise reduction.

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1 Introduction

Since its introduction in the early 90s [20], non-linear diffusion (ND) attracts many researchers not only in image enhancement but also in several applications such as object segmentation [14, 28, 35], motion estimation [23], image compression [33], medical imaging [26], video signal filtering [17], coding-artifact reduction [6], and speckle-noise filtering [4, 34].

ND was derived from the general continuous diffusion equation,

$$\partial_t I = \mathrm{div}(G \cdot \nabla I); \qquad 1 \leq t \leq T, \tag{1}$$

where I is the input image, t is the scale, T is the stopping time, ∇I is the image gradient, div is the divergence operator, and G is the diffusion coefficient. Depending on G, we differentiate *linear diffusion*, where G is a constant and (1) reduces to the heat equation, and *non-linear diffusion*, where G is a function of the image content.

A direct simple discretization of (1), based on Euler scheme, is that of Perona and Malik [20],

$$I_s^{n+1} = I_s^n + \lambda \sum_{p \in W} G(\nabla I_p^n, \sigma) \cdot \nabla I_p^n; \qquad 1 \leq n \leq N, \tag{2}$$

where s is the center pixel, n is the current scale, N is the number of iterations, I_s^n is the image intensity at s and n, I_s^{n+1} is the next-scale version of I^n, I^0 is the input noisy image, and $\nabla I_p = (I_p - I_s)$ is the image gradient in direction $p \in W$. $|W|$ is the number of nearest perpendicular neighbors along which ND is computed; higher number of neighbors can also be used [11].

$G(\nabla I_p, \sigma)$ is the edge-stopping function in direction p.$G(\cdot)$ should approach zero when at edges, i.e., ∇I_p high, and approach one when at homogeneous areas, i.e., $\nabla I_p \rightarrow 0$. σ is the edge strength to control the shape of $G(\cdot)$.

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