

# Area spectrum of the Kerr-Bolt black hole via modified adiabatic invariant quantity

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**Abstract** In this paper, the expression of the adiabatic invariant quantity is presented in the dragged–Painlevé coordinate system. Via revisited adiabatic invariant quantity, using Bohr–Sommerfeld quantization rule and the first law of the black hole thermodynamics, we derive the spectroscopy of the black hole. The result shows that the area and entropy spectra are respectively equally spaced and independent of black hole parameters and the area spectrum of the black hole is  $\Delta A = 8\pi l_p^2$ , which confirms the initial proposal of Bekenstein, and there is no need to impose the small angular momentum limit in contrast to the quasinormal mode method.

**Keywords** Area spectrum · Kerr-Bolt black hole · Adiabatic invariant quantity · Bohr–Sommerfeld quantization rule

## 1 Introduction

The study of spectroscopy from black holes via invariant quantity is a hot spot of current topics in black hole physics. It is widely believed that the horizon area of a black hole in equilibrium has a discrete and equally spaced spectroscopy, i.e.  $A_n = \gamma l_p^2 n$ , where  $n = 1, 2, 3, \dots, l_p$  is the plank length and  $\gamma$  is regarded as a numerical coefficient of the order of unity. There have been many attempts to discuss black hole spectroscopy in various ways (Bekenstein 1973, 1974; Hod 1998; Kunstatter 2003; Maggiore 2008; Medved 2008, 2009; Ropotenko 2009; Padmanabhan and

Patel 2003; Li 2012a, 2012b; Majhi and Vagenas 2011; Jiang and Han 2012; Zeng and Liu 2012; Setare and Vagenas 2005; Vagenas 2008; Fernando 2009; Kwon and Nam 2010; Wei et al. 2009; Gonzalez et al. 2010; Lopez-Ortega 2009; Myung 2010; Fang 2012; Dreyer 2003; Banerjee et al. 2010; Li et al. 2012, 2013; Li 2012; Li and Lin 2013; Ansari 2008; Akhmedov et al. 2006; Akhmedova et al. 2008). One popular method relying on quasinormal modes was first proposed by Hod (1998). Employing the Bohr’s correspondence principle and the spectrum of the quasinormal mode, Hod suggested the area spacing is  $\Delta A = 4l_p^2 \ln 3$ . In Kunstatter (2003) further confirmed this result based on Hod’s proposal and Bekenstein’s proposal for the adiabaticity of the black hole horizon area. In Maggiore (2008) provided a new interpretation with respect to the quasinormal frequencies. He suggested that the area spectrum is determined by the physical frequency of the harmonic oscillator whose frequencies are complex. According to this new explanation, he derived the equally spaced area spectrum  $\Delta A = 8\pi l_p^2$ , which is different from the result obtained by Hod, but consistent with the semiclassical result of Bekenstein (1973). Applying this new explanation, the area spectra have been investigated from various different black holes (Fernando 2009; Kwon and Nam 2010; Wei et al. 2009; Gonzalez et al. 2010; Lopez-Ortega 2009; Myung 2010). Moreover,  $\gamma = 8\pi$  is the most qualified candidate for a universal area spacing.

In Majhi and Vagenas (2011) proposed another scheme to quantize a black hole. It is noteworthy that there is no use at all of the quasinormal frequencies. They exclusively utilized the statement that the black hole horizon area is an adiabatic invariant and derived an equally spaced area spectrum of a class of a static, spherically symmetric black hole with the model-dependent constant  $\gamma = 8\pi$ . In Jiang and Han (2012) argued that the adiabatic invariant quantity proposed in Majhi and Vagenas (2011) was not canonically invariant. They

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