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ORIGINAL ARTICLE

Solitons and Lie group analysis to an extended quantum Zakharov–Kuznetsov equation

Gang-Wei Wang · Tian-Zhou Xu · Stephen Johnson · Anjan Biswas

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Abstract In this paper, complete geometric symmetry of extended quantum Zakharov–Kuznetsov (QZK) equation are investigated. All of the geometric vector fields for the new extended QZK equation are presented. At the same time, a plethora of exact solutions are obtained by the application of the group theorem. In addition, 1-soliton solution of the extended QZK equation with power law nonlinearity is obtained by the aid of traveling wave hypothesis with the necessary constraints in place for the existence of the soliton.

Keywords Extended quantum Zakharov–Kuznetsov equation · Symmetry groups · Exact solutions · Ansatz method

G.-W. Wang (✉) · T.-Z. Xu
School of Mathematics, Beijing Institute of Technology, Beijing 100081, P.R. China
e-mail: pukai1121@163.com

G.-W. Wang
e-mail: wanggangwei@bit.edu.cn

S. Johnson · A. Biswas
Department of Mathematical Sciences, Delaware State University, Dover, DE 19901-2277, USA

S. Johnson
Lake Forest High School, 5407 Killens Pond Road, Felton, DE 19943, USA

A. Biswas
Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah, Saudi Arabia

1 Introduction

The theory of nonlinear evolution equations (NLEEs) play extremely important roles in several areas of applied physics and mathematical physics (Abdou 2011; Bhrawy et al. 2013; Biswas 2008; Biswas et al. 2011; Bluman and Kumei 1989; Ebadi et al. 2012; Garcia et al. 2005; Ghebache and Tribeche 2013; Ibragimov 1994; Khan and Masood 2008; Lie 1881; Moslem et al. 2007; Olver 1986; Ovsiannikov 1982; Pakzad 2010, 2012; Sabry et al. 2008; Wang et al. 2013; Wazwaz 2008, 2012; Xie et al. 2005; Zakharov and Kuznetsov 1974). Some of the areas where NLEEs are frequently observed are nuclear physics, nonlinear optics, plasma physics, astrophysics, biophysics. Therefore it is imperative to develop the mathematical analysis of these NLEEs in a rigorous manner. There are several mathematical tools that study these NLEEs from an analytic perspective. This paper will employ the classic mathematical analysis that will conduct a rigorous study of the governing equation. This is the Lie symmetry or Lie group analysis. This analysis is a one-time classic immortal tool that is applicable to all types of NLEEs. This integration mechanism extracts several form of important solutions to the equations that are very helpful in all areas of mathematical physics and applied physics.

There are several other integration architectures that are employed to seek solutions to the NLEEs. Some of them are the G'/G-expansion method, Backlund transformation, exp-function approach, simplest equation method and several others. However, as it is said that classic never dies, Lie group analysis is an all-time applicable tool to integrate all forms of NLEEs, such as coupled NLEEs, multi-dimensional NLEEs and several other forms. This paper will apply Lie group analysis to solve an important NLEE, known as the extended QZK equation that arises in the study of astrophysics.