

# A model for binary-binary close encounters and collisions from a dynamical point of view

Martha Alvarez-Ramírez · Mario Medina

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**Abstract** The goal of this paper is to provide a model for binary-binary interactions in star clusters, which is based on simultaneous binary collision of a special case of the one-dimensional 4-body problem where four masses move symmetrically about the center of mass. From the theoretical point of view, the singularity due to binary collisions between point masses can be handled by means of regularization theory. Our main tool is a change of coordinates due to McGehee by which we blow-up the singular set associated to total collision and replace it with an invariant manifold which includes binary and simultaneous binary collisions, and then gain a complete picture of the local behavior of the solutions near to total collision via the homothetic orbit.

**Keywords** Trapezoidal 4-body problem · Symmetric collinear 4-body problem · Regularization · Binary collision · Heteroclinic orbit

## 1 Introduction

The 4-body problem has been studied with the help of geometrical restriction methods to analyze the stability and dynamical evolution of symmetric quadruple systems of stars and exoplanetary systems of two planets orbiting a binary pair of stars that may exist within a star cluster, see among others (Steves and Roy 1998, 2001; Roy and Steves 2000; Széll et al. 2004a, 2004b, 2004c).

Most stars do not interact much with their environment after they leave the cradle of the interstellar cloud they are born in Binney and Tremaine (2008). Some stars are born single, and stay that way throughout their life, although they may have acquired a planetary system during the late stages of their formation (Lada 2006). However, most stars are members of a binary system or an even more higher multiplicity system (triples, quadruples, etc.). In such a system, when two stars are sufficiently close, an entirely new set of physical processes may take place including the transfer of mass from one star to another, and even the spiral-in and eventual merging of two or more stars, see Hilditch (2001).

In star clusters, binary systems become the most important type of bound subsystem since close encounters between single stars and binaries may happen, but close encounters involving two binaries can also take place, see Mikkola (1997) and Heggie and Rasio (1996). It is known that systems consisting of more than two bodies are, in general, unstable (Orlov and Petrova 2000). In star clusters, binary subsystems become the most important subsystems since close encounters of a single star with a binary set may happen, also close encounters of two binary systems of stars are not impossible to occur (Mikkola 1985).

Although binary star evolution is much more complex than single star evolution, it can generally still be studied in isolation from its wider environment (Eggleton 2011). After all, the typical separation between stars in the Solar neighborhood is around 1.3 pc or nearly 270000 AU or  $4.1 \times 10^{13}$  km. The solar diameter is nearly 1400000 km. Therefore the ratio is less than 30 million. The exception to this rule occurs in dense stellar systems such as star clusters, both in the disk of the Galaxy (open clusters) and in the halo or bulge (globular clusters), and also near the Galactic Center and in the nuclei of other galaxies (Heggie and Hut 2003).

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M. Alvarez-Ramírez (✉) · M. Medina  
Departamento de Matemáticas, UAM–Iztapalapa, Iztapalapa,  
09340 Mexico, D.F. Mexico  
e-mail: [mar@xanum.uam.mx](mailto:mar@xanum.uam.mx)

M. Medina  
e-mail: [mvmg@xanum.uam.mx](mailto:mvmg@xanum.uam.mx)