

A simple procedure to extend the Gauss method of determining orbital parameters from three to N points

Taghi Mirtorabi

Received: 23 March 2013 / Accepted: 16 September 2013 / Published online: 5 October 2013
© Springer Science+Business Media Dordrecht 2013

Abstract A simple procedure is developed to determine orbital elements of an object orbiting in a central force field which contribute more than three independent celestial positions. By manipulation of formal three point Gauss method of orbit determination, an initial set of heliocentric state vectors \mathbf{r}_i and $\dot{\mathbf{r}}_i$ is calculated. Then using the fact that the object follows the path that keep the constants of motion unchanged, I derive conserved quantities by applying simple linear regression method on state vectors \mathbf{r}_i and $\dot{\mathbf{r}}_i$. The best orbital plane is fixed by applying an iterative procedure which minimize the variation in magnitude of angular momentum of the orbit. Same procedure is used to fix shape and orientation of the orbit in the plane by minimizing variation in total energy and Laplace Runge Lenz vector. The method is tested using simulated data for a hypothetical planet rotating around the sun.

Keywords Astrometry · Celestial mechanics · Planets and satellites: detection

1 Introduction

Orbit determination is a rather old problem, dating back to late eighteenth century when Laplace developed his solution (Taff 1985) and early nineteenth when Giuseppe Piazzi discovered Ceres and the famous young mathematician, Carl

Friedrich Gauss, developed an efficient method of orbit determination (Forbes 1971) to recover the dwarf planet after its reappearance. Launching Sputnik in 1957 arose the need for orbit determination. Weiffenbach (1960) and Guier and Weiffenbach (1997) used data from Doppler tracking of Sputnik I satellite which formed the basis of the transit system. By development in the performance of camera and radio Doppler tracking systems in 1960s, the orbit precession improved to about 10–20 meter. Advances in laser technology in 1970s have raised the accuracy to few centimeter in altitude. Today orbit precessions are routinely in the 2 to 5 cm range (Tapley et al. 1994).

In response to advances in observational accuracy, orbit determination methods also were improved. Kozai (1959) developed a first order theory using Lagrange's planetary equations. This method was the basis of the Smithsonian Astrophysical Observatory Differential Orbit Improvement Program that was used to analyze very accurate Baker-Nunn camera observations and now used as a basis for NASA GEODYN program for precision geophysics application. Brouwer (1959) adapted the Hill-Brown lunar theory to low-Earth satellite problem and developed a method which uses mean orbital elements and include inclination and eccentricity as power series. Lydanne (1963) modified Brouwer's method to handle the singularities of eccentricity and inclination. Using osculating orbital elements, Kaula (1966) developed a theory in Keplerian orbital element space. Kaula theory incorporated third body, resonance and tidal effects. It did not suffer from singularities and handle more general cases. The theory developed by Brouwer and modified and improved by Lydanne and Kaula is now the basic analytical theory used in astro-dynamical orbit determination codes.

The orbit determination method is now a mature topic which is subjects of many classical text books (Vallado 2007; Danby 1992; Tapley et al. 2005; Taff 1985; Mon-

T. Mirtorabi
Department of Physics, Alzahra University, Tehran, Iran

T. Mirtorabi (✉)
School of Astronomy, Institute for Research in Fundamental Sciences (IPM), Tehran, Iran
e-mail: torabi@ipm.ir