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ORIGINAL ARTICLE

New set of universal functions for the two body-initial value problem

M.A. Sharaf · A.S. Saad

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Abstract In this paper, new set of universal functions, $Y_n(\chi;\alpha)$; based on Goodyear's time transformation formula will be developed analytically and computationally for the two body-initial value problem, where n is non negative integer, χ, new independent variable, a kind of *generalized anomaly*, and α is the reciprocal of the orbit's semi-major axis.

For the analytical developments, the proofs of the linear independence of the Y's functions, differential and recurrence formulae satisfied by them and their relations with the elementary function are given. Full, set of the identities for the Y's functions are listed to serve as a ready reference when need. Exact analytical expressions of the general universal Kepler's equation for the time $t \in [t_\ell, t_s]$, together with some orbital parameters are developed as power series of χ; also some recurrence formulae are given to facilitate their computations. Finally, symbolical series solution of the general Kepler's equation is established, and the literal analytical expressions of the coefficients of the series are listed in Horner form for efficient and stable evaluation.

For the computational developments, algorithm for the evaluation of the Y's functions is constructed. Computational initial value problem in terms of the Y's functions is developed for which an efficient iterative method of arbitrary positive integer order of convergence ≥ 2 is established for the universal Kepler's equation. The method is of dynamic natural in the sense that, on going from one iterative scheme to the subsequent one, only additional instruction is needed. The applications of the method are illustrated by numerical examples of some test orbits of different eccentricities. The numerical results are accurate and efficient.

Keywords Universal initial value problem · Goodyear's time transformation · Universal Kepler's equation · One-point iteration formulae

1 Introduction

The applications of the conventional equations of space dynamic for the motion of Earth's artificial satellites give inaccurate predictions. The reason for this is due to the fact that the equations of motion are unstable in the Lyapunov sense (Stiefel and Scheifele 1971). In brief the deficiency of these equations is due to the choice of the variables, which in turn has led some authors to propose successful methods to change of the dependent and/or independent variables so as to regularize the differential equations of motion. On the other hand, besides the known types of conic motion, the given type of an orbit is occasionally changed by perturbing forces acting during finite interval of time. Moreover, a complete interplanetary transfer all types of the two body motion (elliptic, parabolic, or hyperbolic) appear. For examples the escape from the departure planet and the capture by the target planet involve hyperbolic orbits, while the intermediate stage of the mission commonly depicted as a heliocentric ellipse, may also be heliocentric parabola or hyperbola. Thus

M.A. Sharaf
Department of Astronomy, Faculty of Science, King Abdulaziz University, Jeddah, Saudi Arabia
e-mail: Sharaf_adel@hotmail.com

A.S. Saad (✉)
Department of Astronomy, National Research Institute of Astronomy and Geophysics, Cairo, Egypt
e-mail: rmthaan@qu.edu.sa

Present address:
A.S. Saad
Department of Mathematics, Preparatory Year, Qassim University, Buraidah, Saudi Arabia
e-mail: saad6511@gmail.com