

# Thermal relic abundance and anisotropy due to modified gravity

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**Abstract** Alternative cosmologies, based on extensions of General Relativity, predict modified thermal histories in the early universe during the pre Big Bang Nucleosynthesis (BBN) era. When the expansion rate is enhanced with respect to the standard case, thermal relics typically decouple with larger relic abundances. In this paper, we study the dynamical evolution of an  $f(R)$  model of gravity in a homogeneous and anisotropic background which is given by a Bianchi type-I model of the universe filled with dark matter, which is described by a perfect fluid with a barotropic equation of state. As an example of a consistent analysis of modified gravity, we apply the formalism to a simple background solution of  $R + \beta R^n$  gravity. Our analysis shows that  $f(R)$  cosmology allows dark matter masses lesser than 100 GeV, in the regime  $\rho^c \lesssim \rho^m$ . We finally discuss how these limits apply to some specific realizations of standard cosmologies: an  $f(R)$  gravity model, Einstein frame model.

**Keywords** Anisotropic universe ·  $f(R)$  gravity · The Hubble enhancement function · Dark matter

## 1 Introductions

The discovery of the accelerated expansion of the universe (Reiss et al. 1998; Perlmutter et al. 1998) has motivated the

developments of many models of gravity. These models are built up, typically, either in the framework of the conventional General Relativity or in the framework of its possible generalizations or modifications. In the last years, among the different approaches proposed to generalize Einstein's General Relativity, the so called  $f(R)$ -theories of gravity received a growing attention. The reason relies on the fact that they allow to explain, via gravitational dynamics, the observed accelerating phase of the universe, without invoking exotic matter as sources of dark matter. An alternative approach is extended Einstein's theory of gravity, that  $f(R)$  gravity is one of the extended theories of gravity candidate, which itself was advocated to account for this accelerating universe (Dvali and Turner 2002; Nojiri and Odintsov 2003, 2004; Arkani-Hamed et al. 2004; Capozziello et al. 2003, 2006; Carroll 2005; Abdalla et al. 2005; Aghamohammadi et al. 2009, 2010; Aghamohammadi and saaidi 2011; Khodam-Mohammadi et al. 2012; Saaidi et al. 2010). This kind of theories is fruitful and economics with respect to other theories. In this view the effective Lagrangian,  $L$ , in the early universe includes higher order curvature terms and inflation may be a natural result of this theory. In Starobinsky (1980), it was shown that involving a term proportional to the square of the scalar curvature, i.e.,  $L = R + \beta R^2$ , results in a quasi-de Sitter expansion. In this model the Hubble parameter decreases slowly for large  $a$  before going into an oscillation phase which can reheat the universe (Mijic et al. 1986; Anderson and Suen 1987). In principle, one can assume that the effective Lagrangian is a function of the scalar curvature  $L = f(R)$  (Barrow and Ottewill 1983; Barrow and Cotsakis 1988, 1989). The time dependence of the scale factor depends on  $f(R)$ , e.g., if one chooses  $f(R) = R + \beta R^2 + \gamma R^3$ , instead of an exponential like inflation, he may obtain a power law expansion for the universe (Berkin and Maeda 1990; Liddle and Mellor 1992;

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