Astrophys Space Sci
DOI 10.1007/s10509-013-1756-z

ORIGINAL ARTICLE

Reduction the secular solution to periodic solution in the generalized restricted three-body problem

Elbaz I. Abouelmagd · M.E. Awad · E.M.A. Elzayat · Ibrahim A. Abbas

Received: 12 October 2013 / Accepted: 16 December 2013

Abstract The aim of the present work is to find the secular solution around the triangular equilibrium points and reduce it to the periodic solution in the frame work of the generalized restricted thee-body problem. This model is generalized in sense that both the primaries are oblate and radiating as well as the gravitational potential from a belt. We show that the linearized equation of motion of the infinitesimal body around the triangular equilibrium points has a secular solution when the value of mass ratio equals the critical mass value. Moreover, we reduce this solution to periodic solution, as well as some numerical and graphical investigations for the effects of the perturbed forces are introduced. This model can be used to examine the existence of a dust particle near the triangular points of an oblate and radiating binary stars system surrounded by a belt.

E.I. Abouelmagd (✉) · I.A. Abbas
Mathematics Department, Faculty of Science and Arts (Khulais), King Abdulaziz University, Jeddah, Saudi Arabia
e-mail: eabouelmagd@kau.edu.sa

I.A. Abbas
e-mail: ibrabbas7@yahoo.com

E.I. Abouelmagd
e-mail: eabouelmagd@gmail.com

M.E. Awad
Department of Space Science, Faculty of Science, Cairo University, Cairo, Egypt

E.M.A. Elzayat
Mathematics Department, Faculty of Science and Arts (Khulais—Girls Branch), King Abdulaziz University, Jeddah, Saudi Arabia

Keywords Restricted three-body problem · Secular and periodic solutions · Oblateness coefficients · Radiation pressure · Potential from the belt

1 Introduction

The problem of three bodies in its most general form means that the three participating bodies are free to move in space and initially move in any given manner under the influence of a given force field. The significance of this problem in space dynamics will appear when the bodies move under the influence of their mutual gravitational attraction according to the Newtonian Law of gravitation. This law specifies that attractive forces between each pair of masses are inversely proportional to the squares of their distances and are proportional to the product masses of the respective particles.

A first consequence of this Law comes when two of the bodies approach each other such that the separation distance between them goes to zero and the force between them also comes to infinity. This circumstance is called double or triple collision according to whether two or three of the participating particles go to the same position in space at the same time. A second consequence of the force law, it follows that when one of the three participating particles is very smaller than the other two. In this situation the motion of the two larger particles will not be influenced by the smaller particle. This dynamical system is referred to as the restricted three-body problem. Therefore, if the motion of the smallest particle is found, we can determine the motion of the other two particles by setting the mass of the smaller particle as zero.

From the above discussion the restricted problem is an abstraction in the physical sense and an approximation in the mathematical sense. Since there is no effect for the smaller

Published online: 28 December 2013