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ORIGINAL ARTICLE

Orbital evolution dynamics of two satellites in encounter phase using multiple scales expansion

S.E. Abd El-Bar

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Abstract An approximate solution of the encounter problem of two small satellites describing initially elliptical orbits around a massive oblate primary is obtained. The equations of motion of the center of mass of the two masses are developed in the most general form without any restrictions on the orbital elements. The method of multiple scales which seeks a solution whose behavior depends on several time scales is used. To overcome the singularity the equations of motion are transformed to the Struble variables. An analytical second order theory of the evolution dynamics is obtained. A MATHEMATICA program is constructed. The evolution dynamics of the orbital parameters between the perturbed and the unperturbed cases are plotted. The effect of changing eccentricity and changing inclination on the orbital parameters are highlighted.

Keywords Encounter problem · Multiple variables expansion · Struble variables

1 Introduction

The encounter problem is usually defined as follows (e.g. Petit and Henon 1986): consider two light bodies m_2 and m_3 describing initially coplanar and circular orbits, with slightly different radii, around a heavy spherical massive m_1. Also, the primary is not spherical but oblate. As the distance between m_2 and m_3 gets smaller their mutual attraction is no longer negligible and becomes comparable to that of each to the primary and we say that we have an encounter problem. The measure of the difference in semi-major axes at the encounter time is called the impact parameter. The problem may be more generalized such that the orbits be elliptical (e.g. Moons et al. 1988) and not coplanar (e.g. Brumberg and Ivanova 1990).

There are very well known areas of astronomy and celestial mechanics problem incorporates the phenomena of encounter, e.g. rings around the planets and Planetesimals, coorbital motion, the accretion disks, the temporary capture of comets, the distribution of particles around the Earth, the crowded region occupied by communication satellites, and in interplanetary missions.

The most famous method of solution of the encounter problem is the multiple scales. It seeks a solution whose behavior depends on several time scales, e.g. the different frequencies in the dynamical system (in a decreasing order of magnitude). The method is a generalized form of the LindStedt-Poincaré technique which is based on the observation that the frequency of a nonlinear oscillation may depend upon its amplitude. The frequency is expanded in an asymptotic series, and the coefficients of each term in the series are determined in such a way the solution is free from secular terms. In a similar fashion, the method of multiple scales allows the solution to vary on fast and slow time scales. The main difference is that multiple scales assumes that the coefficients of each scale are fixed, and uses a variation-of-parameters approach and lets the constants of integration that appear in the linear solution be functions that vary on the slow time scale. This procedure results in systems of partial differential equations at each order that should be solved to produce a uniform solution of the problem.

S.E. Abd El-Bar (✉)
Department of Mathematics, Faculty of Science, Tanta University, Tanta 31527, Egypt
e-mail: so_abdelbar@yahoo.com

Present address:
S.E. Abd El-Bar
Department of Math., Faculty of Science, Taibah University, Medina, Kingdom of Saudi Arabia

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