Astrophys Space Sci (2013) 348:591–598
DOI 10.1007/s10509-013-1599-7

ORIGINAL ARTICLE

Local Lorentz transformation and exact solution in $f(T)$ gravity theories

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Received: 21 July 2013 / Accepted: 8 August 2013 / Published online: 28 September 2013

Abstract A *general tetrad fields*, with an arbitrary function of radial coordinate, preserving spherical symmetry, is provided. Such tetrad is split into two matrices: The first matrix represents a Local Lorentz Transformation (LLT), which contains an arbitrary function. The second matrix represents a proper tetrad fields which satisfy the field equations of $f(T)$ gravitational theory. This *general tetrad* is applied to the field equations of $f(T)$. We derive a solution with one constant of integration to the resulting field equations of $f(T)$. This solution gives a vanishing value of the scalar torsion. We calculate the energy associated with this solution to investigate what is the nature of the constant of integration.

Keywords $f(T)$ theory of gravity · Local Lorentz transformation · Exact solution · Energy

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1 Introduction

Nowadays many researchers believed that our universe is suffering an accelerated expansion. Recent observations data from type Ia supernovae (Perlmutter et al. 1998, 1999; Reiess et al. 1998) in associated with large scale structure (Abazajian et al. 2004) and cosmic microwave background anisotropies (Spergel et al. 2003) have shown main proof for such cosmic acceleration. It seems that we need some energy components, i.e. dark energy, having negative pressure to be responsible for this late-time acceleration (Copeland et al. 2006). One of the main hot topic in modern theoretical cosmology, is to understand the behavior of dark energy. An alternative method to understand dark energy is to amend the General Relativity (GR) theory on the large scales. Among these modified theories, scalar-tensor theories (Perrotta et al. 2000; Boisseau et al. 2000), $f(R)$ gravity (Kleinert and Schmidt 2002; Capozziello et al. 2003; Nojiri and Odintsov 2009; Sotiriou and Faraoni 2010; Setare 2008; Setare and Mohammadipour 2013) which are studied extensively.

The Teleparallel Equivalence of General Relativity (TEGR) (Hehl et al. 1976; Mikhail and Wanas 1977; Møller 1978; Hayashi and Shirafuji 1979, 1981; Hayashi 1977; Pellegrini and Plebański 1963; Flanagan and Rosenthal 2007; Geng et al. 2011, 2012; Bamba et al. 2012; Wu and Geng 2012a, 2012b; Gu et al. 2013; Geng and Wu 2013) which introduced by Albert Einstein (Einstein 1928, 1930) is a one way to look at the theory beyond general relativity (GR). Teleparallel gravity (TG) uses the Weitzenböck connection Γ, which has no curvature R but has torsion T, is different from GR which uses the Levi-Civita connection {}. An amended gravity has been proposed by extending T in the TG action to be an arbitrary function $f(T)$ to explain the late-time acceleration of the universe. In $f(T)$ gravitational theory, torsion is responsible for the acceleration