Astrophys Space Sci (2013) 348:377–383
DOI 10.1007/s10509-013-1577-0

ORIGINAL ARTICLE

Region of motion in the Sitnikov four-body problem when the fourth mass is finite

L.P. Pandey · I. Ahmad

Received: 14 May 2013 / Accepted: 18 July 2013 / Published online: 14 August 2013

Abstract This article deals with the region of motion in the Sitnikov four-body problem where three bodies (called primaries) of equal masses fixed at the vertices of an equilateral triangle. Fourth mass which is finite confined to moves only along a line perpendicular to the instantaneous plane of the motions of the primaries. Contrary to the Sitnikov problem with one massless body the primaries are moving in non-Keplerian orbits about their centre of mass. It is investigated that for very small range of energy h the motion is possible only in small region of phase space. Condition of bounded motions has been derived. We have explored the structure of phase space with the help of properly chosen surfaces of section. Poincarè surfaces of section for the energy range $-0.480 \le h \le -0.345$ have been computed. We have chosen the plane (q_1, p_1) as surface of section, with q_1 is the distance of a primary from the centre of mass. We plot the respective points when the fourth body crosses the plane $q_2 = 0$. For low energy the central fixed point is stable but for higher value of energy splits in to an unstable and two stable fixed points. The central unstable fixed point once again splits for higher energy into a stable and three unstable fixed points. It is found that at $h = -0.345$ the whole phase space is filled with chaotic orbits.

Keywords Sitnikov four-body problem · Poincarè surface of section · Phase space · Chaotic orbits

1 Introduction

The two-body problem is one of the last problem about which so much can be said. The addition of even one more body to the problem increases the possible complexity of the resulting dynamics without bound. If one desires a qualitative description of three-body dynamics then restricting to particular configuration of three masses is often the only way to proceed. One popular configuration is known as Sitnikov's Problem. In this problem two bodies (called primaries) of equal masses revolve around their common centre of mass in circular/elliptic orbits. A third infinitesimal mass which does not influence the dynamics of the primaries confined to move only along a line perpendicular to the plane of motions of the primaries and passes through the centre of mass of the primaries.

It is important to recall some significant studies made by many scientists/mathematicians. Pavanini (1907) was the first who describe this dynamical model. Mac Millan (1913) presented it as an example of an integrable system of the restricted three-body problem. The elliptic Sitnikov problem becomes important when Sitnikov (1960) himself presented the problem to show the existence of oscillatory motions in the three-body problem. Moser (1973) has proved the existence of the chaotic orbits. Perdios and Markellos (1988) have studied the stability and bifurcation of the Sitnikov motion and has shown the bifurcations in the case of unequal primaries. Hagel (1992) has examined the problem by a new analytic approach, and have also shown that it is valid for bounded small amplitude solution and eccentricities of the primaries. He has linearized the equation in 'z' to obtain the Hill's equation. Dvorak (1993) has investigated "numerical results of the Sitnikov problem" by taking eccentricities for the primaries orbit as $0.33 \le e_{primaries} \le 0.66$. Jalali and Pourtakdoust (1997) have determined the regular and chaotic solutions in the Sitnikov problem near the $3/2$ commensurability. Dvorak and Yi (1997) have studied the extended Sitnikov Problem, where three equal masses stay always in the Sitnikov configuration. They have inves-

L.P. Pandey (✉) · I. Ahmad
Department of Mathematics, Jamia Millia Islamia, New Delhi, India
e-mail: lppandey16april@gmail.com