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Charged static axial symmetric solutions and scalar structures

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Abstract This paper is devoted to the study of static axially symmetric spacetime with anisotropic fluid by means of structure scalars in the presence of electromagnetic field. The structure scalars in terms of physical variables are evaluated through the Einstein-Maxwell field equations and the inhomogeneity factors are identified. We also explore analytic solutions for isotropic as well as anisotropic fluids. It is found that isotropic solution turns out to be a charged solution which has no correspondence with the Weyl metrics while the anisotropic solution has the correspondence with the Weyl metrics.

Keywords Relativistic fluids · Electromagnetic field · Axial symmetry

1 Introduction

In general relativity, the solution of the field equations of static axially symmetric source is referred to the Weyl metrics. It is well-known that Schwarzschild solution is the only static and asymptotically flat vacuum solution which has a regular horizon (Israel 1976). Zipoy-Voorhees solution belongs to the family of the Weyl metrics and has a correspondence with the Schwarzschild metric (Zipoy 1966; Voorhees 1970). This is the only solution for which the physical components of the Riemann tensor does not contain singularity. For all other Weyl solutions (there are as many distinct Weyl solutions as there are distinct harmonic functions), these components contain singularity (Weyl 1917,1919; Stephani et al. 2003). A natural question arises which is the better exact vacuum solution to describe the deviation from spherical to non-spherical axially symmetric static source? There is no unique answer to this question.

In the study of self-gravitating objects, the spherical symmetry plays an important role to describe the occurrence of white dwarfs, neutron stars and black holes. However, in the presence of strong gravitational fields, it is essential to deviate from spherical to non-spherical symmetries to find exact solutions. It is well-known that different spacetimes may satisfy the field equations for different physically meaningful energy-momentum tensors. Herrera and his collaborators (Herrera et al. 2000, 2001a, 2003; Herrera 2005) used the M–Q spacetime and γ-metric as a source of Weyl solutions to illustrate deviations to non-spherical symmetry.

Tiwari et al. (1991) have discussed static axially symmetric (cylindrical coordinates) charged dust and found a new class of electromagnetic mass models. Tiwari and Ray generalized this work for static spherical symmetric spacetime (1991a) and for static axially symmetric spacetime (spherical coordinates) (1991b). Regardless of the symmetry conditions, they concluded that static charged dust distribution is of purely electromagnetic origin. Similarly, the electromagnetic mass models were obtained for the static Levi-Civita axially symmetric spacetime (Ray et al. 1993). They concluded that any charged dust source is of electromagnetic origin.

Many investigations are devoted to understand the interaction between electromagnetic and gravitational fields. Bekenstein (1971) was the first who extended the work from neutral to charge case by generalizing Oppenheimer-Volkoff equations (Oppenheimer and Volkoff 1939). Since then a large amount of work has been done in this sce-

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