## ORIGINAL ARTICLE

## Sitnikov restricted four-body problem with radiation pressure

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**Abstract** An analytical study of the elliptic Sitnikov restricted four-body problem when all the primaries as source of same radiation pressure is presented. We find a solution, which is valid for small bounded oscillations in case of moderate eccentricity of the primary. We have linearized the equation of motion to obtain the Hill's type equation. Using the Courant and Snyder transformation, Hill's equation transformed into harmonic oscillator type equation. We have used the Lindstedt-Poincare perturbation method and again we have applied the Courant and Snyder transformation to obtain the final result.

**Keywords** Sitnikov problem · Four-body problem · Lindstedt-Poincare method · Perturbation theory · Radiation pressure

## 1 Introduction

In the present paper we have studied the Sitnikov problem extended to four-body problem when all the primaries are source of same radiation pressure. The Sitnikov problem is a special case of the restricted three-body problem where the two primaries of equal masses ( $m_1 = m_2 = m = 1/2$ ) are moving in circular or elliptic orbit around their centre

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M.R. Hassan Department of Mathematics, S.M. College, Bhagalpur 812007, India e-mail: hassansmc@gmail.com of mass under Newtonian force of attraction and the third body of mass  $m_3$  (the mass of the third body is much less than the mass of the primaries) moves along the line which is passing through the centre of mass of the primaries and is perpendicular to the plane of motion of the primaries. We are extending this problem to 4-bodies.

The importance of radiation influence on celestial bodies has been studied by many scientists (Simmons et al. 1985; Ragos and Zafiropoulos 1995; Bhatnagar and Chawla 1979; Elipe and Lara 1997; Perdiou et al. 2012). Sitnikov (1960) has shown the existence of oscillating motion of the third body. Sitnikov problem has been studied by many scientists. Perdios and Markellos (1988) have studied stability and bifurcation of Sitnikov motion. Liu and Sun (1990) have studied the mapping instead of the original differential equation and discovered that there exist a hyperbolic invariant set. Hagel (1992) has studied the problem by a new analytical approach. It is valid for bounded small amplitude solution and eccentricities of the primaries. Belbruno et al. (1994) have studied the family of periodic orbits which bifurcate from the circular Sitnikov motion. Faruque (2002) has studied the axial oscillation of a planetoid in the restricted three body problem: The circular Sitnikov problem. Faruque (2003) has established the new analytical expression for the position of the infinitesimal body in the elliptic Sitnikov problem. His solution is valid for small bounded oscillation in case of moderate primary eccentricities. Perdios (2007) has studied the manifolds of families of 3D periodic orbits associated to Sitnikov motions in the restricted three-body problem. Perdios et al. (2008) has studied the straight-line oscillations of the Sitnikov family of the photogravitational circular restricted three-body problem as well as the associated families of three-dimensional periodic orbits. They have computed accurately 49 critical orbits at which families of 3D periodic orbits of the same period bi-