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ORIGINAL ARTICLE

Stability regions of equilibrium points in restricted four-body problem with oblateness effects

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Abstract In this paper, we extend the basic model of the restricted four-body problem introducing two bigger dominant primaries m_1 and m_2 as oblate spheroids when masses of the two primary bodies (m_2 and m_3) are equal. The aim of this study is to investigate the use of zero velocity surfaces and the Poincaré surfaces of section to determine the possible allowed boundary regions and the stability orbit of the equilibrium points. According to different values of Jacobi constant C, we can determine boundary region where the particle can move in possible permitted zones. The stability regions of the equilibrium points expanded due to presence of oblateness coefficient and various values of C, whereas for certain range of t ($100 \leq t \leq 200$), orbits form a shape of cote's spiral. For different values of oblateness parameters A_1 ($0 < A_1 < 1$) and A_2 ($0 < A_2 < 1$), we obtain two collinear and six non-collinear equilibrium points. The non-collinear equilibrium points are stable when the mass parameter μ lies in the interval $(0.0190637, 0.647603)$. However, basins of attraction are constructed with the help of Newton Raphson method to demonstrate the convergence as well as divergence region of the equilibrium points. The nature of basins of attraction of the equilibrium points are less effected in presence and absence of oblateness coefficients A_1 and A_2 respectively in the proposed model.

Keywords Restricted four-body problem · Poincaré surface of section · Oblateness · Equilibrium points · Basins of attraction

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1 Introduction

To study the motion of celestial bodies, restricted four-body problem is one of the important problem in the dynamical system. An application of the restricted four-body problem is illustrated in the general behavior of the synchronous orbit in presence of the Moon as well as the Sun whereas coupled restricted three-body problem is one of the example of restricted four-body problem. The problem is restricted in the sense that one of the masses is taken to be small, that the gravitational effect on the other masses by the fourth mass is negligible. The smaller body is known as infinitesimal mass (body) and remaining three finite massive bodies called primaries.

The classical restricted four-body problem may be generalized to include different types of effect such as oblateness coefficient, radiation pressure force, Poynting-Robertson drag etc. Various authors have studied the restricted four-body problem and examined the existence of equilibrium points such as Hadjidemetriou (1980), Michalodimitrakis (1981), Kalvouridis et al. (2007) and Papadakis (2007). Further, Baltagiannis and Papadakis (2011b) discussed the equilibrium points and their stability in the restricted four-body problem.

On the other hand, in recent years many perturbing forces, such as oblateness, radiation forces of the primaries, Coriolis and centrifugal force, variation of the masses of the primaries etc. have been included in the study of restricted three-body problem (RTBP). The RTBP with oblate effect has been studied by many investigators such as Sharma and Rao (1975), Abouelmagd and El-Shaboury (2012), Khanna and Bhatnagar (1999), Douskos (2011) etc.

Determination of the stability regions of the infinitesimal body was introduced by Poincaré (1892) during the study of periodic orbit of the system. This is very good technique

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