

# A five dimensional Kaluza-Klein bulk viscous string cosmological model in Brans-Dicke scalar-tensor theory of gravitation

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Received: 3 May 2013 / Accepted: 16 May 2013 / Published online: 1 June 2013  
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**Abstract** In this paper, we have investigated a five dimensional Kaluza-Klein space-time in the frame work of Brans-Dicke (Phys. Rev. 124:925, 1961) scalar-tensor theory of gravitation when the source of energy momentum tensor is a bulk viscous fluid containing one dimensional cosmic strings. We have obtained a determinate solution of the field equations using the special law of variation for Hubble's parameter proposed by Bermann (Nuovo Cimento B 74:182, 1983) We have also used a barotropic equation of state for the pressure and density. Some physical properties of the model are also discussed.

**Keywords** Kaluza-Klein model · Bulk viscosity · Cosmic strings · Brans-Dicke theory

## 1 Introduction

There has been a considerable interest in constructing cosmological models in alternative theories of gravity during past decades. The study of cosmological models in Brans and Dicke (1961) and Saez and Ballester (1986) scalar-tensor theories of gravitation is quite important in view of the fact that scalar fields play a vital role in inflationary cosmology. Brans-Dicke theory introduces a long range scalar

field  $\varphi$  interacting equally with all forms matter (with the exception of electromagnetism) besides the metric tensor  $g_{ij}$  and a dimensionless coupling constant  $\omega$ . In Saez-Ballester scalar-tensor theory the metric is coupled with a dimensionless scalar field in a simple manner. This theory suggests a possible way to solve missing matter problem in non-flat FRW cosmologies. The field equations given by Brans and Dicke for the combined scalar and tensor fields are

$$\square\varphi = \varphi_{;k}{}^k = \frac{8\pi}{3+2\omega}T \quad (1)$$

$$R_{ij} - \frac{1}{2}g_{ij}R = -\frac{8\pi}{\varphi}T_{ij} - \frac{\omega}{\varphi^2}\left(\varphi_{,i}\varphi_{,j} - \frac{1}{2}\varphi_{,k}\varphi^{,k}\right) - \frac{1}{\varphi}(\varphi_{;ij} - g_{ij}\square\varphi) \quad (2)$$

where  $\varphi$  is the scalar field,  $\omega$  is the dimensionless coupling constant (this should be constrained as  $\omega \geq 40,000$  for its consistency with solar system bounds, Bertotti et al. 2003; Felice et al. 2006),  $T_{ij}$  is the energy momentum tensor and  $R_{ij}$  and  $R$  have their usual meaning. Also,

$$T_{;j}{}^{ij} = 0 \quad (3)$$

is a consequence of the field equations (1) and (2). Here semicolon indicates covariant derivative and comma denotes ordinary derivative with respect to  $x^k$ .

Brans-Dicke (BD) theory of gravity is one of the most important scalar-tensor theories due to its vast cosmological implications (Bertolami and Martins 2000; Banerjee and Pavon 2001). Several aspects of BD cosmology have been investigated by many authors. Singh and Rai (1983) presented a nice review of work done in BD theory. It is well known that spatially homogeneous and anisotropic

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