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ORIGINAL ARTICLE

On the periodic solutions of a rigid dumbbell satellite in a circular orbit

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Abstract The aim of the present paper is to provide sufficient conditions for the existence of periodic solutions of the perturbed attitude dynamics of a rigid dumbbell satellite in a circular orbit.

Keywords Dumbbell satellite · Periodic orbits · Averaging theory

1 Introduction and statement of the main results

In this paper we consider the attitude dynamics, perturbed by small torques, of a rigid dumbbell satellite in a circular orbit under the gravitational torque of a central Newtonian force field. Our objective is to provide sufficient conditions for the existence of periodic motions about the satellite's center of mass that are asymptotic to translational motion in an absolute coordinate system. This type of motion, denoted, *cylindrical equilibrium*, is well known in the astrophysics literature on satellite's dynamics (see for instance Guirao et al. 2013; Vera 2009, 2010).

These motions have an important application to satellite orientation problems because a satellite can reach some specified nominal regime along periodic trajectories only through the influence of gravitational torques and other small perturbed torques induced by some control mechanism.

Following the methods developed in (Vera 2010), the equations of motion governing the attitude dynamics of a rigid dumbbell satellite are

$$\begin{aligned}
&\frac{d^2\theta}{dt^2} - 2\frac{d\phi}{dt}\left(1+\frac{d\theta}{dt}\right)\tan\phi + 3\sin\theta\cos\theta \\
&\quad = \varepsilon F_1^*\left(t,\theta,\frac{d\theta}{dt},\phi,\frac{d\phi}{dt}\right), \\
&\frac{d^2\phi}{dt^2} + \left(\left(1+\frac{d\theta}{dt}\right)^2 + 3\cos^2\theta\right)\sin\phi\cos\phi \\
&\quad = \varepsilon F_2^*\left(t,\theta,\frac{d\theta}{dt},\phi,\frac{d\phi}{dt}\right),
\end{aligned} \tag{1}$$

with θ and ϕ the Eulerian angles of nutation and precession. The perturbed torques F_i^*, are smooth functions periodic in the variable t with

$$F_1^*\left(t,0,\frac{d\theta}{dt},0,\frac{d\phi}{dt}\right) \equiv 0, \qquad F_2^*\left(t,0,\frac{d\theta}{dt},0,\frac{d\phi}{dt}\right) \equiv 0,$$

and ε a small real parameter. In this work we are interested in the periodic functions emerging from the equilibrium solution $\theta = 0$ and $\phi = 0$ of (1) when $\varepsilon \to 0$.

By means of the change of coordinates $x = \theta$ and $y = \phi$ and linearizing the equations of (1) in this equilibrium we

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