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ORIGINAL ARTICLE

Generalized uncertainty principle and Bekenstein-Hawking entropy in tunneling rate of Kerr black hole

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Abstract We study the effects of the generalized uncertainty principle in the tunneling formalism for Hawking radiation to evaluate the quantum-corrected Hawking temperature and entropy for a Kerr black hole. By assumption of a spatially flat universe accompanied with expansion of metric, the modified area and entropy of Kerr black hole are calculated and we could obtain an expression for entropy of black hole that is changing with respect to time and Bekenstein-Hawking temperature.

Keywords Kerr black hole · Hawking temperature · B-H entropy · Tunneling probability

1 Introduction

Consider the line element of a Kerr black hole in the accelerated expanding universe that describes the metric of its expansion (Perlmutter et al. 1999; Garnavich et al. 1998; Riess et al. 1998)

$$\begin{aligned} ds^2 = -\left(1-\frac{r_s r}{\rho^2}\right)c^2dt^2 + a(t)^2\biggl(\frac{\rho^2}{\Delta(1-kr^2)}dr^2 \\ + \rho^2 d\theta^2 + \left(r^2+\alpha^2+\frac{r_s r\alpha^2}{\rho^2}\right)\sin^2\theta d\varphi^2 \\ - \frac{2r_s r\alpha^2}{\rho^2}\sin^2\theta d\theta d\varphi\biggr), \end{aligned} \tag{1}$$

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where $r_s = 2GM/c^2$, $\alpha = J/Mc$, $\rho^2 = r^2 + \alpha^2\cos^2\theta$ and $\Delta = r^2 - r_s r + \alpha^2$. Also, k denotes the curvature of space $k = 0, 1, -1$ for flat, closed and open universe respectively and $\mathbf{H}$ is the Hubble parameter, where $\mathbf{H} = \dot{a}/a$. We select $k = 0$ which means that our universe is flat. The Kerr metric, like Schwarzschild's, has an event horizon that is spherical in shape, and its surface area A is given by choosing $\hbar = 1$ and $G = 1$

$$A = 4\pi\left(r_+^2 + \alpha^2\right), \tag{2}$$

whereas its radiuses are given by

$$r_\pm = \frac{1}{2}\left[r_g \pm \sqrt{r_g^2 + 4\alpha^2}\right], \tag{3}$$

where $r_g = 2GM/c^2$. We notice that the area A differs from the Euclidean formula, and this is due to the fact that the geometry of the black hole is non-Euclidean. However, since $r_- < r_+$, the outside observer is concerned only with r_+.

In considering the energy that could be released by interaction with black holes, Stephen Hawking discovered an important theorem in 1971. This, the area theorem, states that in the interactions involving black holes, the total surface area of the event horizon of a black hole can never decrease (in the absence of quantum effects); it can, at the best, remains unchanged. Now let us use this area theorem to estimate the energy extraction limits. For an uncharged Kerr black hole, the horizon area A is

$$A = \frac{8\pi G^2M^2}{c^4}\left[1 + \sqrt{\left(\frac{Jc}{GM^2}\right)^2}\right], \tag{4}$$

which can be calculated from the Kerr space-time metric.This reduces to the area of a Schwarzschild black hole $A = 16\pi G^2M^2/c^4$ that is the largest. For a maximally rotating Kerr black hole $J = GM^2/c$, and $A = 8\pi G^2M^2/c^4$. In Wentzel-Kramers-Brillouin (WKB) approximation, it can