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ORIGINAL ARTICLE

Periodic orbits in the restricted four-body problem with two equal masses

Jaime Burgos-García · Joaquín Delgado

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Abstract The restricted (equilateral) four-body problem consists of three bodies of masses m_1, m_2 and m_3 (called primaries) lying in a Lagrangian configuration of the three-body problem i.e., they remain fixed at the apices of an equilateral triangle in a rotating coordinate system. A massless fourth body moves under the Newtonian gravitation law due to the three primaries; as in the restricted three-body problem (R3BP), the fourth mass does not affect the motion of the three primaries. In this paper we explore symmetric periodic orbits of the restricted four-body problem (R4BP) for the case of two equal masses where they satisfy approximately the Routh's critical value. We will classify them in nine families of periodic orbits. We offer an exhaustive study of each family and the stability of each of them.

Keywords Periodic orbits · Four-body problem · Stability · Characteristic curves · Asymptotic orbits

1 Introduction

Few bodies problems have been studied for long time in celestial mechanics, either as simplified models of more complex planetary systems or as benchmark models where new mathematical theories can be tested. The three–body problem has been source of inspiration and study in Celestial Mechanics since Newton and Euler. In recent years it has been discovered multiple stellar systems such as double stars and triple systems. The restricted three body problem (R3BP) has demonstrated to be a good model of several systems in our solar system such as the Sun-Jupiter-Asteroid system, and with less accuracy the Sun-Earth-Moon system. In analogy with the R3BP, in this paper we study a restricted problem of four bodies consisting of three primaries moving in circular orbits keeping an equilateral triangle configuration and a massless particle moving under the gravitational attraction of the primaries. Here we focus on the study of families of periodic orbits. We refer to this as the restricted four body problem (R4BP). There exist some preliminary studies of this problem in different versions, Simó (1978), Leandro (2006), Pedersen (1944) and Baltagiannis and Papadakis (2011) studied the equilibrium points and their stability of this problem. Other authors have studied the case where the primaries form a collinear configuration. At the time of writing this paper we became aware of the paper Baltagiannis and Papadakis (2011), where they performed a numerical study similar to ours for two cases depending on the masses of the primaries: (a) three equal masses and (b) two equal masses. It is the second case that our work is related to Baltagiannis and Papadakis (2011), although we use a slightly different value of the mass parameter. The reason is the same as the cited authors, of having the primaries moving in linearly stable circular orbits for a value of the mass parameter less but approximately equal to Routh's critical value. By historical and theoretical aspects, we use the same letters used in the Copenhagen category of the R3BP to denote the families of periodic orbits, see Szebehely (1967). The families g, f, a, m, r_2, g_4, g_6 are similar to those families denoted by the same letter in the R3BP, i.e., the family of direct periodic orbits around the mass m_1 of this paper is denoted by the letter g as it was done in the Copenhagen category for each family, but the families j and j_2 are exclusive of this problem because they do not have similar families in the R3BP. Our results confirm and extend four families of periodic orbits found in Baltagiannis and

J. Burgos-García · J. Delgado (✉)
Departamento de Matemáticas, UAM-Iztapalapa, Av. San Rafael Atlixco 186, Col. Vicentina, C.P. 09340 México, D.F., México
e-mail: jdf@xanum.uam.mx