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ORIGINAL ARTICLE

Out-of-plane equilibrium points and their stability in the Robe's problem with oblateness and triaxiality

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Abstract This paper examines the existence and stability of the out-of-plane equilibrium points of a third body of infinitesimal mass when the equations of motion are written in the three dimensional form under the set up of the Robe's circular restricted three-body problem, in which the hydrostatic equilibrium figure of the first primary is an oblate spheroid and the second one is a triaxial rigid body under the full buoyancy force of the fluid. The existence of the out of orbital plane equilibrium points lying on the xz-plane is noticed. These points are however unstable in the linear sense.

Keywords Robe's problem · Out of plane equilibrium points · Buoyancy force · Oblateness · Triaxiality · Stability

1 Introduction

A new kind of restricted three-body problem was formulated by Robe (1977). In this problem, one of the primaries of mass m_1, is a rigid spherical shell, filled with homogeneous and incompressible fluid of density ρ_1, while the second mass m_2 is a small mass point outside the shell and moving around the first primary in a Keplerian orbit, and the infinitesimal mass m_3 is a small sphere of density ρ_3, moving inside the shell and is subject to the attraction of m_2 and the buoyancy force due to the fluid. The radius of m_3 is assumed to be infinitesimal. Robe (1977) discussed only the linear stability of an equilibrium point. In his study, he assumed that the pressure field of the fluid ρ_1 has a spherical symmetry around the center of the shell and he took into account the component of the pressure field which is due to the own gravitational field of the fluid in the shell. This problem may be used to study the problem of small oscillations of the Earth's core in the gravitational field of the Earth-Moon system. Because of its applications, further studies of finding other equilibrium points and their stability under different assumptions have been carried out by some researchers.

Hallan and Rana (2001) investigated the existence of all equilibrium points and their stability in the Robe's (1977) restricted three-body problem. It was seen that the Robe's elliptic restricted three-body problem has only one equilibrium point for all values of the density parameter K and the mass parameter μ, while the Robe's circular restricted three-body problem can have two, three or an infinite number of equilibrium points. They confirmed that the equilibrium point at the center of the shell is stable, whereas triangular and circular points are always unstable. The equilibrium point collinear with the center of the shell and the second primary was found to be stable under some conditions. Hallan and Mangang (2007) studied the Robe's (1977) restricted three-body problem by considering the full buoyancy force and assumed that the hydrostatic equilibrium figure of the first primary is an oblate spheroid.

In notion, the participating bodies in the classical restricted three-body problem are considered to be perfect spheres, but in reality such bodies as Saturn and Jupiter which are not strictly spherical in shape has led to new discovery concerning dynamics predictions in the restricted three-body problem. Global studies of problems with oblateness and triaxiality have been carried out by many researchers, a few of which are Subba Rao and Sharma (1975), Singh and Ishwar (1999), Sharma et al. (2001), Douskos and

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