GPS Solut (2013) 17:309–326
DOI 10.1007/s10291-012-0280-x

ORIGINAL ARTICLE

Analytical 2D GNSS PVT solutions from a hyperbolic positioning approach

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Received: 21 April 2011 / Accepted: 10 July 2012 / Published online: 9 September 2012

Abstract GNSS receivers estimate 3D antenna position and receiver clock bias when at least four satellites are tracked. If only three satellites are available, a 2D antenna position solution is still possible. We derive an almost exact algorithm for the determination of two possible antenna positions and the corresponding receiver clock biases based on pseudorange measurements to three GNSS satellites and a height measurement. The two ambiguous solutions exactly reflect the same height measurement. One of the solutions can be eliminated if some prior knowledge of the user position, for example, near the Earth, is available. In general, a less accurate height measurement gives a less accurate 2D GNSS solution, and vice versa. The determination of the receiver antenna position is based upon the intersection of two confocal hyperboloid sheets and the ellipsoid, resulting in a hyperbola along which the user is located. The algorithm is verified by numerical computations.

Keywords 2D GNSS PVT solution · Hyperbolic positioning · Altitude measurement · Pseudorange differences · Hyperboloid sheets

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Introduction

For 3D user position and receiver clock bias estimation with a global navigation satellite system (GNSS), simultaneous pseudorange measurements from the user antenna to at least four satellites in view are required in general. Situations may occur where only five satellites are visible in the sky at the same time. If in such cases the line-of-sight from receiver antenna to some of the satellites is blocked by objects, it can readily occur that only signals from less than four satellites can be tracked. Then, a GNSS 3D position solution for a GNSS receiver is no longer possible, and therefore a loosely coupled IMU/GNSS navigation system would be unaided.

When less than four satellites are available, partial information can be exploited by using a tightly coupled IMU/GNSS integration scheme, that is, estimation of receiver clock bias and drift in the navigation filter and observation of GNSS pseudoranges or range rates. When a tightly coupled IMU/GNSS navigation system is aided by height, it can be experienced that just three satellites are sufficient for the full observability of space–time, independent of the assumed GNSS receiver clock noise (Fig. 1).

We derive an analytical solution using three GNSS pseudorange measurements and a height (above WGS84 ellipsoid) measurement, which is commonly known as 2D GNSS solution. In high altitudes, the height measurements from a barometric altimeter or, in low altitudes, the height above ground level from a radar altimeter together with terrain height from a map could be used for that purpose. A 2D GNSS solution is commonly provided by GNSS receivers and could be used, for example, to aid a loosely coupled IMU/GNSS navigation system instead of aiding with a height measurement only. We want to emphasize the