## ORIGINAL ARTICLE

## Stability analysis of GPS carrier tracking loops by phase margin approach

Tian Jin · JunXing Ren

Received: 21 March 2012/Accepted: 6 September 2012/Published online: 30 September 2012 © Springer-Verlag 2012

**Abstract** Stability, which is significantly related to the loop parameters, is an important factor in the traditional GPS tracking loop design. Through the analysis of phase margin values in the discrete GPS PLL tacking loop, we are able to theoretically reveal the relationship between loop stability, equivalent noise bandwidth  $B_n$ , predetection integration time T, and loop parameters. We calculate the theoretical limitations for  $B_nT$ , that is, the product of equivalent noise bandwidth multiplied by predetection integration time, for second- and third-order phase-locked loop, respectively. The results are verified by actual data from GPS receivers.

**Keywords** Stability · Phase margin · Phase-locked loop (PLL)

## Introduction

In modern communication systems, phase-locked loop (PLL) technology is widely used in carrier synchronization recovery. With the rapid development of digital electronics technology, much research have been carried out in the Digital PLL field (Lindsey and Chak 1981). The majority of this research was focused on the design of the Phase Detector (John and Brian 2006; Weinfeld and Bar-David 1995) and the loop performance (Zhuang 1996; Zhuang

T. Jin (🖂) · J. Ren

J. Ren e-mail: junxingr@126.com and Tranquilla 1995). However, there is a lack of in-depth research on the stability design of loop filters.

There are currently three methodologies used to design the digital tracking loop filters for GNSS receivers: they are the transformation method, the controlled-root method, and the minimization method. The transformation method is the most commonly used methodology. In this case, analog filter theory is used for the filter design followed by the transformation from a continuous-time system into a discrete-time system. This methodology is popular for tracking of GNSS signals because it is the most simple to implement (Ward 1996; Spilker 1996; Stephens 2001; Tsui 2000; Ward et al. 2006). In the controlled-root method (Stephens and Thomas 1995), the loop filter parameters are determined specifically for each  $B_nT$ , where  $B_n$  is the equivalent noise bandwidth and T is predetection integration time. Designing filters directly in the Z-domain overcomes the deficiencies of the continuous-update approximation when  $B_nT$  is large. The minimization method was introduced by Gupta (1968) using Z-transform and modified Z-transform for analog-digital phase-locked loops. The phase-locked loop is the same as in the continuous case except that the filter is replaced by a discrete filter followed by a hold circuit. This minimization method has been adapted for use in a GNSS receiver loop filter (Kazemi 2008). The minimization technique was used to determine the filter structure and coefficients to design the digital tracking loop directly in the Z-domain based on the linear model of the digital phase lock loop. The loop parameters were determined in order to minimize the variance of the phase error resulting in an optimum loop filter with the design intent to go beyond the  $B_nT$  limits of the other two methods.

The  $B_nT$  value was preliminarily discussed in prior papers (Humphreys et al. 2005). Then, a theoretical

School of Electronic and Information Engineering, BeiHang University, Beijing, People's Republic of China e-mail: jintian@buaa.edu.cn