SHORT COMMUNICATION

A general condition for spontaneous capillary flow in uniform cross-section microchannels

Jean Berthier · Kenneth A. Brakke · Erwin Berthier

Received: 19 February 2013 / Accepted: 13 October 2013 © Springer-Verlag Berlin Heidelberg 2013

Abstract Spontaneous capillary flow (SCF) is a powerful method for moving fluids at the microscale. In modern biotechnology, composite channels—sometimes open—are increasingly used. The ability to predict the occurrence of a SCF is a necessity. In this work, using the Gibbs free energy, we derive a general condition for the establishment of SCF in any composite microchannel of constant cross section, i.e., a microchannel comprising different wall materials and even open parts. It is shown that SCF occurs when the Cassie angle is smaller than $\pi/2$ ($\theta^* < \pi/2$). For a homogeneous confined channel, this relation collapses to the well-known hydrophilic contact angle $\theta < \pi/2$.

Keywords SCF (spontaneous capillary flow) · Gibbs thermodynamic equation · Cassie law

1 Introduction

In modern biotechnology, composite channels—sometimes partly open or with apertures—are increasingly used and spontaneous capillary flow (SCF) is a convenient method to move fluids in such geometries. Spontaneous capillary flow occurs when a liquid volume is moved spontaneously by

J. Berthier (🖂)

CEA-Leti, 17 Avenue des Martyrs, 38054 Grenoble, France e-mail: jean.berthier@cea.fr

K. A. Brakke

Mathematics Department, Susquehanna University, 514 University Avenue, Selinsgrove, PA 17870, USA

E. Berthier

Department of Medical Microbiology and Immunology, University of Wisconsin-Madison, Madison, WI 53705, USA the effect of capillary forces—without the help of auxiliary devices such as pumps or syringes. These devices using SCF are especially useful for portable systems, which greatly benefit to be equipment-free. Geometries facilitating the establishment of capillary flows in confined chanexperimentally and numerically nels have been investigated (Juncker 2002; Kitron-Belinkov et al. 2007; Zimmerman et al. 2007, 2008; Chen et al. 2009). A general law for determining the condition for a SCF in composite microchannels is needed. A first approach has been proposed by Berthier and Brakke (2012) for an open channel of uniform cross section. In this paper, we generalize this former approach and present a theoretical model-based on the Gibbs free energy-to derive the condition for SCF in any composite channels of uniform cross section, i.e., microchannels with non-homogeneous walls and partly open to external air, as sketched in the Fig. 1. We show that the condition for SCF is simply that the generalized Cassie angle for the composite surface be smaller than 90°. Let us recall that the generalized Cassie angle θ^* is the average contact angle defined in the appropriate way, i.e.,

$$\cos\theta^* = \sum_i \left(\cos\theta_i f_i\right) \tag{1}$$

where θ_i is the Young's contact angle with each component *i* (including air for the open parts), and f_i the areal fraction of each component *i* in a cross section of the flow (Fig. 1).

The areal fractions are $f_i = w_i / (\sum_i w_i + w_F)$ and $f_F = w_F / (\sum_i w_i + w_F)$

2 Theory

Our starting point is the Gibbs thermodynamic equation (Gibbs 1873)