Microfluid Nanofluid (2013) 15:701–713
DOI 10.1007/s10404-013-1173-1

RESEARCH PAPER

On the conditional superiority of counter-current over co-current extraction in microchannels

J. R. Picardo · S. Pushpavanam

Received: 17 December 2012 / Accepted: 6 March 2013 / Published online: 10 April 2013
© Springer-Verlag Berlin Heidelberg 2013

Abstract In liquid–liquid extraction, counter-current flow of the phases always results in an improved performance as compared to co-current flow, under similar operating conditions. However, it is challenging to implement counter-current flow in a microchannel. Therefore, the improvement in extraction performance must be significant to justify the selection of counter-current flow over co-current flow in microchannels. In this study, we identify the range of fluid properties and operating conditions for which counter-current operation exhibits significant benefits. For this, simplified mathematical models are developed for both co-current and counter-current extraction in the stratified flow regime. These models, while being simple, capture the essential physics of the extraction process and facilitate a thorough investigation of the relative extraction performance across the parameter space. An analytical solution, based on the theory of Sturm–Liouville linear operators, is obtained for the case of co-current flow. The counter-current model belongs to the class of two-way diffusion equations for which a novel semi-analytical solution is presented. The analysis of the predictions of the models shows that the relative extraction performance is governed by a general principle of *maximum gain at mediocre performance*. These results help identify the significantly restricted range of operating parameters for which counter-current operation is a truly attractive alternative to the co-current mode of extraction in microchannels.

J. R. Picardo · S. Pushpavanam (✉)
Department of Chemical Engineering,
Indian Institute of Technology Madras,
Chennai 600036, India
e-mail: spush@iitm.ac.in

Keywords Mathematical modeling · Two-way diffusion · Microchannel · Extraction · Co-current · Counter-current · Stratified

List of symbols

y	Coordinate along the channel width
x	Coordinate along the channel length
i	Index: 1 denotes carrier phase and 2 denotes solvent phase
c_{i}	Concentration of solute in each fluid phase
v_{i}	Plug flow velocities of each fluid phase
D_i	Diffusivity of solute in each fluid phase
h	Location of the interface
H	Width of the channel
h_{r}	Location of the interface as a fraction of total channel width
K	Equilibrium coefficient of solute concentration
C_{in}	Initial concentration in entering carrier stream
Pe_i	Peclet number in each fluid phase (Eq. 8 for the co-current problem and Eq. 16 for the counter-current problem as well as parameter studies)
β	Ratio of diffusivities
$\varphi_{i,n}(y)$	nth eigen function in each fluid phase (co-current problem)
λ_n	nth eigen value (co-current problem)
ER	Extraction ratio (Eq. 37)
RER	Relative extraction ratio (Eq. 38)

1 Introduction

The extraction of a solute from one liquid phase (carrier) to another (solvent) is a key downstream operation in chemical process industries. The flow of two immiscible liquids