

# R-function Theory for Bending Problem of Shallow Spherical Shells with Polygonal Boundary 

Shanqing Li ${ }^{\text {a }}$, Hong Yuan ${ }^{\text {a }^{*}}$, Xiongfei Yang ${ }^{\text {a }}$, Huanliang Zhang ${ }^{\text {a }}$, Qifeng Peng ${ }^{\text {a }}$<br>${ }^{a}$ MOE Key Laboratory of Disaster Forecast and Control in Engineering, School of Mechanics and Construction Engineering, Institute of Applied Mechanics, Jinan University, Guangzhou 510632, China.

Received 31 October 2019; Accepted 07 January 2020


#### Abstract

The governing differential equations of the bending problem of simply supported shallow spherical shells on Winkler foundation are simplified to an independent equation of radial deflection. The independent equation of radial deflection is decomposed to two Laplace operators by intermediate variable. The R -function theory is applied to describe a shallow spherical shell on Winkler foundation with concave boundary, and then a quasi-Green's function is established by using the fundamental solution and the normalized boundary equation. The quasi-Green's function satisfies the homogeneous boundary condition of the problem. The Laplace operators of the problem are reduced to two simultaneous Fredholm integral equations of the second kind by the Green's formula. The singularity of the kernel of the integral equation is eliminated by choosing a suitable form of the normalized boundary equation. The integral equations are discretized into the homogeneous linear algebraic equations to proceed numerical computing. The singular term in the discrete equation is eliminated by the integral method. Some numerical examples are given to verify the validity of the proposed method in calculating simple boundary conditions and polygonal boundary conditions. A comparison with the ANSYS finite element (FEM) solution shows a good agreement, and it demonstrates the feasibility and efficiency of the present method.


Keywords: Green's Function; R-function; Integral Equation; Bending of Shallow Spherical Shell; Concave Boundary.

## 1. Introduction

As a kind of structural forms, the shells and plates are widely used in various fields, such as, in the large-span roof, the underground foundation engineering, the hydraulic engineering, the large container manufacturing, the aviation, the shipbuilding, the missiles, the space technology, the chemical industry, and so on. Only few problems of the shells and plates with a regular geometric boundary and a simple differential equation can be solved with an analytical or a half analytical method. For most these problems with geometry of arbitrary shape and a complex boundary condition, only numerical methods can be used to solve the problems, such as the boundary element method [1], the Finite Element Method [2] and the finite difference method [3].

In the present paper, the R-function theory and the quasi-Green's function method (QGFM) proposed by Rvachev [4] are utilized. The bending problem of simply supported dodecagon shallow spherical shells on Winkler foundation with concave boundary is studied. The governing differential equation of the problem is decomposed into two simultaneous differential equations of lower order by utilizing an intermediate variable. A quasi-Green's function is established by using the fundamental solution and the boundary equation of the problem. This function satisfies the

[^0]
[^0]:    * Corresponding author: tyuanhong@jnu.edu.cnhttp://dx.doi.org/10.28991/cej-2020-03091487
    

