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Approximation of the Multidimensional Optimal Control Problem for the Heat Equation (Applicable to Computational Fluid Dynamics (CFD))

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Abstract

This work is devoted to finding an estimate of the convergence rate of an algorithm for numerically solving the optimal control problem for the three-dimensional heat equation. An important aspect of the work is not only the establishment of convergence of solutions of a sequence of discrete problems to the solution of the original differential problem, but the determination of the order of convergence, which plays a very important role in applications. The paper uses the discretization method of the differential problem and the method of integral estimates. The reduction of a differential multidimensional mixed problem to a difference one is based on the approximation of the desired solution and its derivatives by difference expressions, for which the error of such an approximation is known. The idea of using integral estimates is typical for such problems, but in the multidimensional case significant technical difficulties arise. To estimate errors, we used multidimensional analogues of the integration formula by parts, Friedrichs and Poincare inequalities. The technique used in this work can be applied under some additional assumptions, and for nonlinear multidimensional mixed problems of parabolic type. To find a numerical solution, the variable direction method is used for the difference problem of a parabolic type equation. The resulting algorithm is implemented using program code written in the Python 3.7 programming language.

Keywords: Approximation of a Three-Dimensional Parabolic Problem; Optimal Control; Convergence of the Gradient Method; Integral Estimates; Functional Convergence Estimation, CFD.

1. Introduction

The heat equation is used to find the dependence of the temperature of the medium on the spatial coordinates and time, for given coefficients of heat capacity and heat conductivity. This is a second order partial differential equation, which is a parabolic type equation. Since the need to determine temperatures in the whole space is often absent, when setting the problem, additional conditions are introduced that determine the restrictions on the solution of the problem for a given area. For example, one of these conditions is to set the temperature distribution at the boundary of the region (the Dirichlet problem).

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