Propagation of longitudinal deformation wave along a lifting rope of variable length

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Abstract

The problem of motion of the rope of variable length consists of solving the boundary value problem with a variable boundary for the one-dimensional wave equation. A change of the rope length is caused by the force acting at the upper cross section of the rope. Studying the wave propagation process along the rope is based on the known integro-differential relation. The problem is reduced to solving two ordinary differential equations with a retarded argument that describe the variable length of the rope and the position of its lower end. The value of argument for functions involved in the right-hand side of the equations lag behind the argument value in the left-hand side of the equations by a time interval it takes for a propagation of the deformation wave throughout the current rope length. The problem is solved by using a technique of the sequential continuation of solution in the cyclic mode for each of the equation alternately. A computer realization of this technique presents no problem. A pilot computer program has been developed for solving the problem. Results of the numerical solution are presented in the case that the active force varies with time according to a piecewise linear relation.

1. Introduction

The problem of longitudinal oscillations of the rope of variable length consists of solving the one-dimensional wave equation on the interval of the space coordinate with the variable boundary. A change in the time of the rope length is caused by the force acting at the upper cross section of the rope. The problem has not an analytical solution. Representation of the rope motion equation in the form of integro-differential equation with variable limits of integration has stimulated the use of asymptotic methods for constructing the approximated solution of the problem (Chizh, 1963; Glushko and Chizh, 1966; Goroshko and Savin, 1971). However, the assessment of accuracy of results achievable by the asymptotic methods leads to problems. The motion of the displacement waves in rods of variable length is considered in Ostapenko (2007) on the assumption that a law of change of the rod length is given in advance. The solution is represented as a sum of four functions which are determined as a result of solving the wave equation for different initial and boundary conditions. Another approach based on representation of the rope as a discrete multi-body system with extensible members (Fritzkowski and Kaminski, 2009) is far from a stage of development applicable to the engineering applications. The present paper is devoted to the problem of the motion of a lifting rope of variable length with a concentrated end load under the action of the force at the upper cross section of the rope. Studying the wave propagation process along the rope is based on the known integro-differential relation (Tikhonov and Samarskii, 1963). The problem is reduced to solving two ordinary differential equations that describe the variable rope length and the motion trajectory of the rope end. A length of the current integration interval of the equations is defined by a time interval for which the solution has been calculated in the previous step of the solving process. A technique of the solution continuation from the given time interval to the following one is used. The present approach has been employed for solving problems on the propagation of elastic waves in a rope of the hoist device and in a belt of the conveyor belt (Razdolsky, 1976; Chervonenko et al., 1983).

2. Boundary value problem on a motion of the lifting rope of variable length

Fig. 1 illustrates the scheme of the hoisting machine consisting of a pulley and an elastic lifting rope that carries a concentrated load at its end. The pulley is rotated by an external torque and the rope is wound onto the pulley. A rope length varies owing to winding the rope onto the pulley. The following symbols are used for designation of the source data: $l = \text{initial length of rope}$, $A = \text{area of the rope cross section}$, $q = \text{linear weight}$, $\rho = \text{linear density}$, $r = \text{initial stress at the upper cross section of the rope}$, $E = \text{elasticity modulus}$, $\sigma = \sqrt{EA}/p = \text{propagation velocity of the}$