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ORIGINAL ARTICLE

Existence and stability of triangular points in the restricted three-body problem with numerical applications

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Abstract In this paper, we prove that the locations of the triangular points and their linear stability are affected by the oblateness of the more massive primary in the planar circular restricted three-body problem, considering the effect of oblateness for J_2 and J_4. After that, we show that the triangular points are stable for $0 < \mu < \mu_c$ and unstable when $\mu_c \leq \mu \leq \frac{1}{2}$, where μ_c is the critical mass parameter which depends on the coefficients of oblateness. On the other hand, we produce some numerical values for the positions of the triangular points, μ and μ_c using planets systems in our solar system which emphasis that the range of stability will decrease; however this range sometimes is not affected by the existence of J_4 for some planets systems as in Earth–Moon, Saturn–Phoebe and Uranus–Caliban systems.

Keywords Restricted three-body problem · Stability of triangular points · Oblate spheroid · Critical mass

1 Introduction

It is known that celestial bodies are irregular bodies and cannot be considered as a spherical permanent in the restricted three-body problem, because the shape of the body affects the stability of movement. In most cases the planets and their natural satellites are extended bodies which are far from being considered as spheres. For purpose of more accuracy to be taken in consideration that the objects are tri-axial or oblate spheroid, this problem has wide applications in many astrophysical problems, Trojan asteroids around the triangular points of the Sun-Jupiter system are actually an example, it is applicable directly to our problem, all of this motivated us to produce the current study.

The planar restricted circular three-body problem describes the motion of the third body, which has infinitesimal mass and moving in the gravitational field of two massive bodies in the same plane which are called the primaries, these two bodies revolve around their center of mass in circular orbit under the influence of their mutual gravitational attraction. This problem possesses five points called Lagrangian points, three of them are called the collinear points L_1, L_2 and L_3 are unstable, they lie on the line joining the primaries, the other two are called the triangular points L_4 and L_5 are stable for the mass ratio $\mu \leq 0.038520896505$, Szebehely (1967).

Some studies which are related to the Lagrangian points by considering one or both primaries are oblate spheroids, whose equatorial planes coincide with the plane of motion are discussed by Sharma (1975); Sharma and Subbarao (1975, 1976); Subbarao and Sharma (1997); Bhatnagar et al. (1994) and Markellos et al. (1996), these studies considered the effect of oblateness up to the linear coefficient J_2 only.

Subbarao and Sharma (1975) studied the problem in a synodic coordinates system when the massive primary is an oblate spheroid; they found that the range of stability will decrease.

Bhatnagar and Hallan (1979) studied the locations and the stability in the linear sense of libration points in the restricted three-body problem when there are perturbations in the potentials between the bodies; they observed that there are five libration points when the perturbing functions satisfy certain conditions; the theory is verified in four cases: classical problem; when the more massive is an oblate

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