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LETTER TO THE EDITOR

Comment on the existence of a long range correlation in the geomagnetic disturbance storm time (Dst) index

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Abstract Very recently (Banerjee et al. in Astrophys. Space, doi:1007/s10509-011-0836-1, 2011) the statistics of geomagnetic Disturbance storm (Dst) index have been addressed, and the conclusion from this analysis suggests that the underlying dynamical process can be modeled as a fractional Brownian motion with persistent long-range correlations. In this comment we expose several misconceptions and flaws in the statistical analysis of that work. On the basis of these arguments, the former conclusion should be revisited.

Keywords Solar-terrestrial relations · Solar wind

In Banerjee et al. (2011) the authors make use of some methods for nonlinear time series, including the recently introduced Visibility (Lacasa et al. 2008) and Horizontal Visibility algorithms (Luque et al. 2009) to describe the statistical properties of geomagnetic time series, and conclude accordingly that its structure is compatible with an underlying long-range correlated stochastic process as the dynamical mechanism originating such phenomenon. This is an important statement and therefore it must rely on solid grounds. In this comment we wish to stress some flaws and possible pitfalls that the authors have committed in their statistical analysis, which prevents us from assuming *a priori* the validity of their findings.

According to the Visibility Algorithm (Lacasa et al. 2009), fractional Brownian motion series with Hurst exponent H map into scale free Visibility graphs with a power law degree distribution of the shape $P(k) \sim k^{-\gamma}$, where

$$\gamma(H) = 3 - 2H \qquad (1)$$

The authors initially center their study in the determination of the Hurst exponent of different processes. It must be first stressed that this relation only restricts to the aforementioned kind of non-stationary stochastic processes with self-similar increments. Therefore, it does not apply for instance to random uncorrelated series, such as the ones firstly analyzed by the authors (Fig. 3 and Table 1 in Banerjee et al. 2011). Notice at this point that in Lacasa et al. (2008) it was already pointed out that random uncorrelated series map into visibility graphs with an *exponential* degree distribution, instead of one following a power law relation, and that there is a theorem that precisely proves such a thing within the Horizontal version of the method (Lacasa and Toral 2010) (something that the authors paradoxically seem to be aware of, given the fact that they make use of the Horizontal visibility algorithm further in their study). Note that such exponential relation becomes evident just by visual inspection, in a semi-log plot of $P(k)$. This is presumably the shape behind the results shown in Fig. 3 of Banerjee et al. (2011) (a log-log plot of an exponential function reveals as a curved shape, such as the one found by the authors, and shouldn't be confused with a power law). It is indeed a bit disturbing that the authors claim that the resulting value of $H = 0.2232$ "*coincides with the exact specified value for the perfectly random series*", a straightforwardly wrong statement.

The second time series addressed by the authors is the Conway series. Again, while this is a self-similar series, it does not fall under the umbrella of an fBm (if only because the Conway series is a deterministic fractal series recursively generated, instead of a stochastic one), and accordingly relation (1) cannot be used here as well (Table 1 in Banerjee et al. 2011).

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