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Huge progeny production during the transient of a quasi-species model of viral infection, reproduction and mutation

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ABSTRACT

Eigen's quasi-species model describes viruses as ensembles of different mutants of a high fitness "master" genotype. Mutants are assumed to have lower fitness than the master type, yet they coexist with it forming the quasi-species. When the mutation rate is sufficiently high, the master type no longer survives and gets replaced by a wide range of mutant types, thus destroying the quasi-species. It is the so-called "error catastrophe". But natural selection acts on phenotypes, not genotypes, and huge amounts of genotypes yield the same phenotype. An important consequence of this is the appearance of beneficial mutations which increase the fitness of mutants. A model has been recently proposed to describe quasi-species in the presence of beneficial mutations. This model lacks the error catastrophe of Eigen's model and predicts a steady state in which the viral population grows exponentially. Extinction can only occur if the infectivity of the quasi-species is so low that this exponential is negative. In this work I investigate the transient of this model when infection is started from a small amount of low fitness virions. I prove that, beyond an initial regime where viral population decreases (and can go extinct), the growth of the population is super-exponential. Hence this population quickly becomes so huge that selection due to lack of host cells to be infected begins to act before the steady state is reached. This result suggests that viral infection may widespread before the virus has developed its optimal form.

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1. Introduction

It seems that an unavoidable consequence of the increase in complexity of a system is the appearance of parasites. These are entities able to exploit backdoors, bypasses, holes... of the system for their own benefit, sometimes even at a cost for the system. We see a huge variety of these parasites in biology, ranging from viruses to humans. Society, in fact, is one of those complex systems amenable to exploitation by free-riders (the paradigm of the Public Goods game [1] is but one prominent acknowledgment of the existence of this social parasitism). More recently, the widespread use of computers and the arrival of Internet has made us witness the emergence and proliferation of computer viruses, trojans, worms, spam, phising, and all kinds of forms of parasitism, which flood the web using the same mechanisms aimed at allowing the transmission of information. Apparently, whenever a complex mechanism emerges, it is soon invaded by its specific parasites.

Parasites need not be complex: on the contrary, by being very specific to a particular mechanism, they are able to do their job with very simple tools. Paradigmatic among parasites for their extreme simplicity are viruses. Their success is such that they are the most abundant life forms on Earth [2]. Their existence is an unavoidable outcome of the very evolutionary

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