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On the identities of modulo-p partitions[☆]

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ABSTRACT

Some identities between partitions and compositions were obtained in the literature. As a natural extension, we introduce and study modulo-p partitions, where p is a positive integer. Moreover, several recurrence relations and some sufficient conditions for the existence of modulo-p partitions are given, respectively. In addition, we obtain some identities of modulo-p partitions. In the end, using the properties of a binary tree, we provide a method to determine modulo-p partitions.

1. Introduction

Lots of interesting partition identities occurred since the first identity was given by Euler [1]. On the other hand, research on identities involving partitions and ordered partitions (namely, compositions [2]) are few and have only occurred in recent years [3–5]. The first such effort was made by Agarwal [3] in 2003, and recently, Guo [4] give some other identities using Agarwal's method.

Inspired by the definitions of "odd–even" partitions, "even" partitions and their partition identities in [3,4], we consider a more general problem of the identities between partitions and compositions in this paper.

Throughout this paper, let s, n, p, m be positive integers, and b, q, t, c, r, d be nonnegative integers such that $s = tp + r$, $n = cp + d$, $0 \le b, q, r, d \le p - 1$. Let x^y denote the abbreviation of $\underbrace{x + x + \cdots + x}_{y}$. We first introduce the (p, b, q)-partition, the F-(p, b, q)-partition and the (p, b)-composition as follows. For convenience, they are all called modulo-p partitions.

Definition 1.1 (*[6]*)**.** A two-rowed array of nonnegative integers $\begin{pmatrix} a_1 & a_2 & \cdots & a_k \\ b_1 & b_2 & \cdots & b_k \end{pmatrix}$ is called a Frobenius partition of n, where $a_1 > a_2 > \cdots > a_k \ge 0$, $b_1 > b_2 > \cdots > b_k \ge 0 (k \in Z_+)$, and $n = k + \sum_{i=1}^{k} a_i + \sum_{i=1}^{k} b_i$.

Remark 1. Note that each partition can be represented by a Frobenius notation. For instance, the Frobenius notation of $22 = 8 + 7 + 3 + 3 + 1$ (see the following figure) is $\begin{pmatrix} 7 & 5 & 0 \\ 4 & 2 & 1 \end{pmatrix}$.

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