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ORIGINAL ARTICLE

Homologous gravitational collapse in Lagrangian representation

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Abstract The classical problem of spherical homologous gravitational collapse with a polytropic equation of state with $\gamma = 4/3$ is examined in Lagrangian fluid coordinate. The fluid velocity $v(t) = dr(t)/dt = \eta dy/dt$ is derived from the evolution function $y(t)$ where $\eta = r(0)$ is the radial fluid label in Lagrangian formulation. The evolution function $y(t)$, which describes the collapse time history of a finite pressure cloud, is solved which happens to be identical to the well established parametric form of Mestel (Mon. Not. R. Astron. Soc., 6:161–198, 1965) for cold cloud collapse. The spatial structure is described by a nonlinear equation of the density profile function $q(z)$ with $z = a\eta$. Due to the nonlinearity, the collapse profile is highly non-uniform in space. For moderate values of $q(0)$, the solutions are homologous. For large $q(0)$, homology is broken leading to the formation of a central core and a central cavity. The stellar envelope bounding the central cavity collapses under the additional external gravity of the central core, generating eventually a sequence of cavity-shell structure in the envelope, until the entire mass of the original cloud is accounted for.

Keywords Gravitation · Hydrodynamics · Star: formation

1 Eulerian gravitational collapse

The fundamental issue of gravitational collapse of a large but finite massive gas cloud concerns the star and nebula formations. Bonner (1956) had demonstrated that the equilibrium of a finite isothermal gas cloud under its self-gravity could be unstable as the mass increases. Mestel (1965) solved the collapse equation of a cold cloud using a kinematic approach where the fluid velocity $v(r, t)$ was related to the fluid position $r(t)$ by $v(r, t) = dr(t)/dt$, thus eliminating $v(r, t)$ as a fluid variable, to obtain the celebrated parametric solution. Bodenheimer and Sweigart (1968) studied numerically the evolution sequence of a collapsing gas cloud with finite pressure again with $v = dr/dt$ under different initial density distributions and different surface boundary conditions. Penston (1969) analyzed analytically the cold collapse with a smooth maximum for density in the kinematic approach, and also put forward a self-similar analysis in x of an isothermal collapsing sphere with thermal velocity c where the radial position $r(t)$ was scaled with time t through $x = r(t)/ct$. Following the kinematic approach, Goldreich and Weber (1980) construct similarity solutions of homologous collapse using a time function $a(t) = (t + t_0)^{2/3}$ to scale different fluid variables and arrived at a differential equation that describes the radial structures.

Larson (1969) examined numerically through a set of conservation equations of mass, momentum, and energy in Eulerian representation the formation of a protostar, and presented in Appendix C the isothermal similarity solutions. Shu (1977) studied anew the homologous collapse of an isothermal sphere with the Eulerian conservation equations. He interpreted the singular solutions, where the coefficients of the nonlinear differential equations vanish, and constructed the expansion-wave, inside-out collapse scenario. Hunter (1977) added a new class of isothermal self-similar solutions on previously known ones. Whitworth and Summers (1985) brought to the attention the importance of the stability of the initial isothermal gas cloud and the external driving pressure. These two factors transform each known solution into a continuum.

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