

Generalization of Levi-Civita regularization in the restricted three-body problem

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Abstract A family of polynomial coupled function of n degree is proposed, in order to generalize the Levi-Civita regularization method, in the restricted three-body problem. Analytical relationship between polar radii in the physical plane and in the regularized plane are established; similar for polar angles. As a numerical application, trajectories of the test particle using polynomial functions of 2, 3, ..., 8 degree are obtained. For the polynomial of second degree, the Levi-Civita regularization method is found.

Keywords Celestial mechanics · Regularization · Restricted three-body problem

1 Introduction

The regularization (in Celestial Mechanics) is a transformation of space and time variables, in order to eliminate the singularities occurring in equations of motion. As Szebehely show (see Szebehely 1967), the purpose of regularization is to obtain regular differential equations of motion and not regular solutions.

The regularization was introduced by Levi-Civita in (1906) in plane, and generalized by Kustaanheimo and Stiefel in (1965) in space. At the beginning, the regularization was developed for studying the singularities of Kepler motion, for analyzing the collisions of two point masses, and

for improving the numerical integration of near-collision orbits. Many studies of the regularization problem are in the restricted three-body problem, where there are two singularities. We can regularize local (one of them), or global. Birkhoff (1915), Thiele (1896), Burrau (1906), Lemaître (1955), Arenstorf (1963), Érdi (2004), Szücs-Csillik and Roman (2012), and many other researchers studied the regularization of the restricted three-body problem.

In order to obtain the regularized equations of motion, one introduces a generating function S , which depends on two harmonic and conjugated functions f and g . But there are many harmonic and conjugated functions. Using different couples of polynomial functions, one can obtain different methods of regularization. For the polynomial of second degree we obtain the Levi-Civita regularization method. By consequence, in this article we created a class of regularization methods, which all have in common the idea that f and g are harmonic and conjugate polynomial functions. We studied analytically some properties of these methods.

Starting from the graphical representation of the test particle's trajectory in the circular restricted three-body problem in the physical plane, and imposing a set of initial conditions, we obtained trajectories in the regularized plane, using 7 regularization methods. So, the methods can be compared not only by canonical equations of motion, but also by the shape of the obtained trajectories.

2 The restricted three-body problem

First of all, let us analyze why the regularization is useful in the restricted three-body problem. For simplicity, we shall consider in the following that the third body moves into the orbital plane ($z = 0$). Denoting S_1 and S_2 the components of the binary system (whose masses are m_1 and m_2),

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