Astrophys Space Sci (2014) 349:25–32
DOI 10.1007/s10509-013-1606-z

LETTER

The cosmological constant as an eigenvalue of a Sturm-Liouville problem

Artyom V. Astashenok · Emilio Elizalde · Artyom V. Yurov

Received: 3 August 2013 / Accepted: 18 August 2013 / Published online: 18 September 2013

Abstract It is observed that one of Einstein-Friedmann's equations has formally the aspect of a Sturm-Liouville problem, and that the cosmological constant, Λ, plays thereby the role of spectral parameter (what hints to its connection with the Casimir effect). The subsequent formulation of appropriate boundary conditions leads to a set of admissible values for Λ, considered as eigenvalues of the corresponding linear operator. Simplest boundary conditions are assumed, namely that the eigenfunctions belong to L^2 space, with the result that, when all energy conditions are satisfied, they yield a discrete spectrum for $\Lambda > 0$ and a continuous one for $\Lambda < 0$. A very interesting situation is seen to occur when the discrete spectrum contains only one point: then, there is the possibility to obtain appropriate cosmological conditions without invoking the anthropic principle. This possibility is shown to be realized in cyclic cosmological models, provided the potential of the matter field is similar to the potential of the scalar field. The dynamics of the universe in this case contains a sudden future singularity.

Keywords Dark energy · Cosmological constant · Stourm-Liouville problem

A.V. Astashenok (✉) · A.V. Yurov
Institute of Physics and Technology, Baltic Federal University of I. Kant, 14 Nevsky St., Kaliningrad 236041, Russia
e-mail: artyom.art@gmail.com

E. Elizalde
ICE/CSIC and IEEC, Campus UAB, Facultat de Ciències, Consejo Superior de Investigaciones Científicas, Torre C5-Par-2a pl, 08193 Bellaterra (Barcelona) Spain

1 Introduction

In Einsteinian Gravity, when quantum corrections are taken into account the determination of the cosmological constant (cc), Λ, is very much related with vacuum fluctuations and, eventually, the Casimir effect (Elizalde 2012), in one way or other (Elizalde 2006; Elizalde et al. 2005, 2003). The problem of the cc is one of the most important and intriguing in modern cosmology and QFT. The traditional way to understand why Λ is so small ($\Lambda < 10^{-120}$ in Planck units) is ordinarily based on the anthropic principle (Weinberg 1987). Up to now one has no reasonable alternative way to explain this fact in Einsteinian Gravity, unless one moves on to extended models with dynamical dark energy or modified gravities (Nojiri and Odintsov 2011).

Recently Barrow and Shaw (2011a, 2011b, 2011c) suggested a non-anthropic solution to the problem: by making Λ into a field and restricting the variations of the action with respect to it by causality, they managed to obtain an additional Einstein constraint equation. One can say that this approach is based on a different *interpretation* of the cosmological constant, notably to consider the cosmological constant to be a field variable.

In the present work we use a somehow similar procedure to calculate the value of Λ, namely to interpret it as an eigenvalue of a Sturm-Liouville problem, rather than as an integration constant. Such interpretation for example is offered by Garattini (2006), Capozziello and Garattini (2007), Garattini (2009). Our approach is in fact more conservative than the method in Garattini (2006) since we use the standard Einstein-Friedmann equations in frames of General Relativity *without* any additional constraint equation such as Wheeler-deWitt equation. Instead, we consider one of Friedmann's equations as a spectral problem and look for a class of boundary conditions which may allow us to actually calculate the corresponding eigenvalues