Astrophys Space Sci (2014) 349:125–135
DOI 10.1007/s10509-013-1629-5

ORIGINAL ARTICLE

On the triangular equilibrium points in the photogravitational relativistic restricted three body problem

F.A. Abd El-Salam · S.E. Abd El-Bar

Received: 20 July 2013 / Accepted: 4 September 2013 / Published online: 27 September 2013

Abstract The photogravitational restricted three body within the framework of the post-Newtonian approximation is carried out. The mass of the primaries are assumed changed under the effect of continuous radiation process. The locations of the triangular points are computed. Series forms of these locations are obtained as new analytical results. In order to introduce a semi-analytical view, a Mathematica program is constructed so as to draw the locations of triangular points versus the whole range of the mass ratio μ taking into account the photogravitational effects and/or the relativistic corrections. All the obtained figures are analyzed. The size of relativistic effects of about.08 normalized distance unit is observed.

Keywords Restricted three body problem · Relativistic corrections · Photogravitational effects · Triangular points

F.A. Abd El-Salam (✉)
Department of Astronomy, Faculty of Science, Cairo University, Cairo, 12613, Egypt
e-mail: f.a.abdelsalam@gmail.com

S.E. Abd El-Bar
Department of Mathematics, Faculty of Science, Tanta University, Tanta, Egypt
e-mail: so_abdelbar@yahoo.com

Present address:
F.A. Abd El-Salam
Department of Mathematics, Faculty of Science, Taibah University, Al-Madinah, Saudi Arabia

Present address:
S.E. Abd El-Bar
Department of Applied Mathematics, Faculty of Applied Science, Taibah University, Al-Madinah, Saudi Arabia

1 Introduction

The restricted three-body problem describes the motion of an infinitesimal mass moving under the gravitational effect of the two finite masses, called primaries, which move in circular orbits around their centre of mass on account of their mutual attraction and the infinitesimal mass not influencing the motion of the primaries. It is originally formulated due to approximate circular motion of the planets around the Sun and the small masses of the asteroids and the satellites of the planets compared to the planet's masses.

The photogravitational restricted three-body problem arises from the corresponding classical problem if at least one of the interacting bodies is an intense emitter of radiation. Radzievskii (1950) was the first to formulate and discuss the problem, for three specific bodies: the sun, a planet, and a dust particle. It was found that an allowance for direct solar radiation pressure is a change in the positions of the libration points and the appearance of new libration points in comparison with classical problem (Radzievskii 1953).

As the solar radiation pressure force F_p is exactly opposite to the gravitational attraction force F_g (neglecting a tiny effect of aberration) and changes with the distance by the same law possible to consider that the result of action of this force will lead to reducing the effective mass of the Sun or a particle. It is acceptable to speak about a reduced mass of the particle as effect of reducing its mass depends on the properties of the particle itself.

Thus the particle is acted upon by the Sun's resulting force: $F = F_g - F_p = F_g(1 - F_p/F_g) = qF_g$ where, $q = 1 - F_p/F_g$ is the mass reduction factor constant for given particle which is expressed in terms of particle radius a, density δ and the solar radiation pressure efficiency factor k (in CGS units) $q = 1 - [(5.6 \times 10^{-5})/a\delta]k$. The assumption $q =$ const. is equivalent to neglecting fluctuations in the beam of solar radiation and the effect of the planet's shadow.