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ORIGINAL ARTICLE

Asymptotic behavior of a scalar field with an arbitrary potential trapped on a Randall-Sundrum's braneworld: the effect of a negative dark radiation term on a Bianchi I brane

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Abstract In this work we present a phase space analysis of a quintessence field and a perfect fluid trapped in a Randall-Sundrum's Braneworld of type 2. We consider a homogeneous but anisotropic Bianchi I brane geometry. Moreover, we consider the effect of the projection of the five-dimensional Weyl tensor onto the three-brane in the form of a negative Dark Radiation term. For the treatment of the potential we use the "Method of f-devisers" that allows investigating arbitrary potentials in a phase space. We present general conditions on the potential in order to obtain the stability of standard 4D and non-standard 5D de Sitter solutions, and we provide the stability conditions for both scalar field-matter scaling solutions, scalar field-dark radiation solutions and scalar field-dominated solutions. We find that the shear-dominated solutions are unstable (particularly, contracting shear-dominated solutions are of saddle type). As a main difference with our previous work, the traditionally ever-expanding models could potentially re-collapse due to the negativity of the dark radiation. Additionally, our system admits a large class of static solutions that are of saddle type. These kinds of solutions are important at intermediate stages in the evolution of the universe, since they allow the transition from contracting to expanding models and viceversa. New features of our scenario are the existence of a bounce and a turnaround, which lead to cyclic behavior, that are not allowed in Bianchi I branes with positive dark radiation term. Finally, as specific examples we consider the potentials $V \propto \sinh^{-\alpha}(\beta\phi)$ and $V \propto [\cosh(\xi\phi) - 1]$ which have simple f-devisers.

Keywords Dark energy · Dark radiation · Randall-Sundrum · Bianchi I

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1 Introduction

The idea that our Universe is a brane embedded in a 5-dimensional space, on which the particles of the standard model are confined, while gravity is allowed to propagate not only on the brane but also in the bulk of the higher-dimensional manifold, was proposed by Randall and Sundrum (1999a, 1999b). These braneworld models were baptized as Randall-Sundrum type 1 (RS1) and type 2 (RS2). The original motivation of Randall and Sundrum (1999a) to propose the RS1 scenario was to look for an explanation to the hierarchy problem, whereas the motivation of the RS2 scenario was to propose an alternative mechanism to the Kaluza-Klein compactifications (Randall and Sundrum 1999b).

It is well-known that the cosmological field equations on the RS2 brane are essentially different from the standard 4-dimensional cosmology (Binetruy et al. 2000a, 2000b; Bowcock et al. 2000; Apostolopoulos et al. 2005). In fact, in the