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ORIGINAL ARTICLE

Robe's restricted problem of 2 + 2 bodies when the bigger primary is a Roche ellipsoid and the smaller primary is an oblate body

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Abstract In this problem, one of the primaries of mass m_1 is a Roche ellipsoid filled with a homogeneous incompressible fluid of density ρ_1. The smaller primary of mass m_2 is an oblate body outside the Ellipsoid. The third and the fourth bodies (of mass m_3 and m_4 respectively) are small solid spheres of density ρ_3 and ρ_4 respectively inside the Ellipsoid, with the assumption that the mass and the radius of the third and the fourth body are infinitesimal. We assume that m_2 is describing a circle around m_1. The masses m_3 and m_4 mutually attract each other, do not influence the motions of m_1 and m_2 but are influenced by them. We have extended the Robe's restricted three-body problem to $2+2$ body problem under the assumption that the fluid body assumes the shape of the Roche ellipsoid (Chandrashekhar in Ellipsoidal figures of equilibrium, Chap. 8, Dover, New York, 1987). We have taken into consideration all the three components of the pressure field in deriving the expression for the buoyancy force viz (i) due to the own gravitational field of the fluid (ii) that originating in the attraction of m_2 (iii) that arising from the centrifugal force. In this paper, equilibrium solutions of m_3 and m_4 and their linear stability are analyzed. We have proved that there exist only six equilibrium solutions of the system, provided they lie within the Roche ellipsoid. In a system where the primaries are considered as Earth-Moon and m_3, m_4 as submarines, the equilibrium solutions of m_3 and m_4 respectively when the displacement is given in the direction of x_1-axis or x_2-axis are unstable.

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1 Introduction

Robe (1977) has investigated a new kind of restricted three-body problem in which one of the primaries of mass m_1 is a rigid spherical shell filled with a homogeneous incompressible fluid of density ρ_1. The smaller primary is a mass point m_2 outside the shell. The third body of mass m_3, supposed moving inside the shell, is a small solid sphere of density ρ_3, with the assumption that the mass and the radius of the third body are infinitesimal. He further assumed that the mass m_2 describes a Keplerian orbit around the mass m_1. He has proved that the centre of the first primary, is the only equilibrium solution for all values of the density parameter K, mass parameter μ, eccentricity parameter e. Further, he has discussed the linear stability of this equilibrium solution. He has explicitly discussed two cases. In the first case, the orbit of m_2 around m_1 is circular and in the second case, the orbit is elliptic, but the shell is empty (i.e. no fluid inside it) or the densities of m_1 and m_3 are equal. In each case, the domain of stability has been investigated for the whole range of parameters occurring in the problem.

In the above problem, the existence of only one equilibrium solution namely, centre of the first primary is discussed. Hallan and Rana (2001) studied the existence and the linear stability of all the equilibrium solutions in the Robe's restricted three-body problem. They have proved that besides the centre, there are other equilibrium solutions which exist only when $K \neq 0$ and the second primary moves around the first in a circular orbit. In addition to the existing collinear equilibrium solution, they have shown the ex-