ORIGINAL ARTICLE

Orbits and manifolds near the equilibrium points around a rotating asteroid

Yu Jiang · Hexi Baoyin · Junfeng Li · Hengnian Li

Received: 20 June 2013 / Accepted: 28 August 2013 / Published online: 19 September 2013 © Springer Science+Business Media Dordrecht 2013

Abstract We study the orbits and manifolds near the equilibrium points of a rotating asteroid. The linearised equations of motion relative to the equilibrium points in the gravitational field of a rotating asteroid, the characteristic equation and the stable conditions of the equilibrium points are derived and discussed. First, a new metric is presented to link the orbit and the geodesic of the smooth manifold. Then, using the eigenvalues of the characteristic equation, the equilibrium points are classified into 8 cases. A theorem is presented and proved to describe the structure of the submanifold as well as the stable and unstable behaviours of a massless test particle near the equilibrium points. The linearly stable, the non-resonant unstable, and the resonant equilibrium points are discussed. There are three families of periodic orbits and four families of quasi-periodic orbits near the linearly stable equilibrium point. For the nonresonant unstable equilibrium points, there are four relevant cases; for the periodic orbit and the quasi-periodic orbit, the structures of the submanifold and the subspace near the equilibrium points are studied for each case. For the resonant equilibrium points, the dimension of the resonant manifold is greater than 4, and we find at least one family of periodic orbits near the resonant equilibrium points. As an application of the theory developed here, we study relevant orbits for the asteroids 216 Kleopatra, 1620 Geographos, 4769 Castalia and 6489 Golevka.

Y. Jiang (⊠) · H. Li

Y. Jiang · H. Baoyin (⊠) · J. Li School of Aerospace, Tsinghua University, Beijing 100084, China e-mail: baoyin@tsinghua.edu.cn **Keywords** Asteroids · Equilibrium points · Stability · Periodic orbits · Smooth manifold

1 Introduction

Recently, several sample return missions to Near-Earth Asteroid (NEA) have been selected (Barucci 2012; Barucci et al. 2012a, 2012b; Boynton et al. 2012; Brucato 2012; Duffard et al. 2011), including MarcoPolo-R¹ and OSIRIS-Rex.² MarcoPolo-R is a sample-return mission to a NEA selected for an assessment study in the framework of ESA's Cosmic Vision 2015-25 program (Barucci 2012; Barucci et al. 2012a, 2012b; Brucato 2012). The primary target of the MarcoPolo-R mission is the asteroid (341843) 2008 EV5, which offers an optimal mission profile both from the operational and technical standpoints (Brucato 2012). OSIRIS-Rex is a sample-return mission to asteroid (101955) Bennu that was selected by NASA in May 2011 as the third New Frontiers Mission (Boynton et al. 2012). The analysis of data collected by MarcoPolo-R should help us to understand the geomorphology, dynamics and evolution of a NEA (Barucci et al. 2012a).

Missions like the ones described above demand a detailed study of the dynamics of a spacecraft (which is customarily modelled as a massless test particle) near asteroids. This interesting topic will be the subject of our study. The application of the classical method of the Legendre polynomial series to the gravitational potential of asteroids produces divergencies at some points due to the irregular shape of the asteroid (Balmino 1994; Elipe and Lara 2003). Several strategies have been considered to solve this difficulty.

State Key Laboratory of Astronautic Dynamics, Xi'an Satellite Control Center, Xi'an 710043, China e-mail: jiangyu_xian_china@163.com

¹https://www.oca.eu/MarcoPolo-R/.

²http://osiris-rex.lpl.arizona.edu/.