## Poynting–Robertson (P–R) drag and oblateness effects on motion around the triangular equilibrium points in the photogravitational R3BP

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**Abstract** This paper investigates the motion of a test particle around the triangular equilibrium points under the effects of radiation pressure of the second primary and its Poynting–Robertson (P–R) effect when the first primary is an oblate spheroid. It is seen that triangular points are influenced by the presence of these parameters: radiation pressure from the secondary and the incidental P–R effect and the oblateness of the first primary. The linear stability of the problem is studied and applied to the binary system RXJ 0450.1-5856, the triangular points are unstable due to positive roots in the Lyapunov sense when P–R effect is considered as against their conditional stability in the absence of P–R drag effect.

Keywords Celestial mechanics  $\cdot$  Radiation pressure  $\cdot$  Oblateness  $\cdot$  R3BP

## **1** Introduction

The most celebrated problem of space dynamics is the problem of three bodies (3BP). The problem is defined in terms of three bodies with arbitrary masses attracting one another according to the Newtonian law of gravitation and that are free to move in space. The degenerate case of the 3BP is the restricted three-body problem (R3BP) described as the motion of an infinitesimal mass moving under the gravitational effects of two finite masses, called primaries, which move

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J. Singh e-mail: jgds2004@yahoo.com in circular orbits around their common center of mass on account of their mutual gravitational attraction and the infinitesimal mass not influencing the motion of the primaries. The R3BP is one of the most widely studied areas in space dynamics as well as in celestial mechanics; and very significant results have been produced by well known mathematicians and scientists in an attempt to understand and predict the motion of natural bodies.

The classical restricted three-body problem (CR3BP) does not consider the case when at least one of the interacting bodies is an intense emitter of radiation. In certain stellar dynamics problems, it is altogether inadequate to consider solely gravitational force. For example, when a star acts upon a particle in a cloud of gas and dust, the dominant factor is by no means gravity, but the repulsive force of the radiation pressure (Radzievsky 1950). In this regard, it is reasonable to modify the model by superimposing a light repulsion field whose source coincides with the source of the gravitational field of the main bodies. Several studies (Hamilton and Burns 1992; Singh and Ishwar 1999; Kunitsyn 2000; Singh 2009; Singh and Leke 2010; Singh et al. 2010) of the restricted problem have included a radiation pressure force.

The R3BP assumes that the masses concerned are spherically symmetrical in homogeneous layers, but it is found that celestial bodies, such as Saturn and Jupiter, are sufficiently oblate (Beatty et al. 1999). The minor planets (e.g., Ceres) and meteoroids have irregular shapes (Millis et al. 1987; Norton and Chitwood 2008). The oblateness or triaxiality of a body can produce perturbations-deviations from the two-body motion. The orbit of the fifth satellite of Jupiter, Amalthea, is one of the most striking examples of perturbations arising from oblateness in the solar system (Moulton 1914).

Rotation in stars produces an equatorial bulge due to the centrifugal force, and, as a result of the rapid rotation after