ORIGINAL ARTICLE

The singly averaged elliptical restricted three-body problem

S.M. Elshaboury · A. Mostafa

Abstract The plane, singly averaged, elliptical restricted three body problem is considered in the article. The first three terms are taken in the perturbing function. The equations of motion in terms of the canonical elements of Delaunay are obtained. And the change of the elements of motion of the satellite due to the perturbing function is calculated. An application is given in the case of a satellite in the earth-moon system.

1 Introduction

Various averaged variants of the restricted three-body problem have appeared in celestial mechanics in connection with the study of secular long-period and resonance perturbations. With time each of these variants has become independent problem of celestial mechanics with important applications.

The importance of these model problems for astronomy is determined by two circumstances. First, in expansions of the perturbing functions of these problems one retains the most important terms determining the motion over long time intervals. Second, the differential equations of motion of the enumerated problems are integrated in quadratures. All this makes it possible to conduct a whole series of interesting investigations. The orbits obtained in these problems can serve as the intermediate orbits in the construction of theories of motion of celestial bodies in the most difficult cases. For example, Vashkov'yak and Teslenko (2008) analyzed the periodic evolving orbits of the singly averaged Hill problem. Voyatzis et al. (2012) studied the stability and regions of existence of periodic orbits in the elliptic Hill Problem. Şelaru and Cucu-Dumitrescu (1995) presented a method of obtaining asymptotic approximations for motions near a Lagrange point in the planar, elliptic, restricted three-body problem by using a von Zeipel-type method. Tupikova (2009) construct an averaging procedure for the motion of a massless body around the central body perturbed by fully interacting planets. Elshaboury and Mostafa (2012) studied the translational rotational motion of an earth satellite using the averaging technique based on Lie series.

Aksenov (1979a, 1979b) studied the doubly averaged, elliptical, restricted three-body problem.

In this work the plane, singly averaged, elliptical restricted three body problem is studied in terms of the canonical elements of Delaunay. The perturbing function is considered up to order $(r/r')^3$. The case of a satellite in the earth-moon system is studied and numerical results are given.

2 Perturbing function

As is known, the elliptical, restricted, three-body problem consists of the following. Let a body P' with a mass m' revolve in an elliptical orbit about a body P_0 with a mass m_0. Then let there be a body P with a very small mass such that it is attracted by bodies P_0 and P' but does not have any effect on their motion. It is required to study the motion of body P.

S.M. Elshaboury · A. Mostafa
Department of Mathematics, Faculty of Science, Ain Shams
University, Cairo, Egypt

A. Mostafa (✉)
Department of Mathematics, The German University in Cairo,
Cairo, Egypt
e-mail: ahmed.mahmoud@guc.edu.eg