# Resonant periodic orbits in the exoplanetary systems 

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#### Abstract

The planetary dynamics of 4/3, 3/2, 5/2, 3/1 and $4 / 1$ mean motion resonances is studied by using the model of the general three body problem in a rotating frame and by determining families of periodic orbits for each resonance. Both planar and spatial cases are examined. In the spatial problem, families of periodic orbits are obtained after analytical continuation of vertical critical orbits. The linear stability of orbits is also examined. Concerning initial conditions nearby stable periodic orbits, we obtain long-term planetary stability, while unstable orbits are associated with chaotic evolution that destabilizes the planetary system. Stable periodic orbits are of particular importance in planetary dynamics, since they can host real planetary systems. We found stable orbits up to $60^{\circ}$ of mutual planetary inclination, but in most families, the stability does not exceed $20^{\circ}-30^{\circ}$, depending on the planetary mass ratio. Most of these orbits are very eccentric. Stable inclined circular orbits or orbits of low eccentricity were found in the $4 / 3$ and $5 / 2$ resonance, respectively.


Keywords Extrasolar planets • General three body problem • Mean motion resonances • Periodic orbits • Planetary systems

## 1 Introduction

Over the last decades, there has been a tremendous increase of extrasolar planetary systems discoveries. Many of such

[^0]systems consist of more than one planet and the study of planetary orbits concerning their long term stability is very interesting. Also, many planets seem to be locked in mean motion resonance (MMR), with the majority of which in $2 / 1$ and by descending order, in $3 / 2,5 / 2,3 / 1,4 / 1,4 / 3,5 / 1$ and $7 / 2$. However, the present estimation of their initial conditions may change significantly after obtaining additional observational data in the future.

Our study refers to the dynamics of a two-planet system locked in a MMR. Michtchenko et al. (2006) had studied the dynamics of many MMR in the planar case by extracting an appropriate averaged Hamiltonian and computing the families of its stationary points. Modeling a two-planet system with the general three body problem (GTBP), we can study the dynamics of the non-averaged system by computing families of periodic orbits in a suitable rotating frame (Voyatzis and Hadjidemetriou 2005; Hadjidemetriou 2006; Voyatzis 2008; Voyatzis et al. 2009). These families of periodic orbits should coincide with the families of stationary points, provided that the averaged Hamiltonian is sufficiently correct. Also, it has been shown that families of periodic orbits can constitute paths that can drive the migration process and, consequently, trap the planets in a MMR (Lee and Peale 2002; Ferraz-Mello et al. 2003; Hadjidemetriou and Voyatzis 2010).

All of the above mentioned studies refer to the planar problem. In this paper, we, also, present new results for the planar case but we, mainly, focus on the dynamics of planetary orbits in space. The spatial GTBP, where planets have a mutual inclination, has been studied only for the $2 / 1$ resonance in Antoniadou and Voyatzis (2013). We herewith determine families of symmetric periodic orbits in all possible configurations of the above mentioned resonances. We compute the spatial families by analytic continuation of vertical critical orbits (v.c.o.) of the planar general problem; a


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