Astrophys Space Sci
DOI 10.1007/s10509-013-1662-4

ORIGINAL ARTICLE

Stability of equilibrium points in a CR3BP with oblate bodies enclosed by a circular cluster of material points

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Received: 9 April 2013 / Accepted: 2 October 2013

Abstract We have investigated an improved version of the classic restricted three-body problem where both primaries are considered oblate and are enclosed by a homogeneous circular planar cluster of material points centered at the mass center of the system. In this dynamical model we have examined the effect on the number and on the linear stability of the equilibrium locations of the small particle due to both, the primaries' oblateness and the potential created by the circular cluster. We have drawn the zero-velocity surfaces and we have found that in addition to the usual five Lagrangian equilibrium points of the classic restricted three-body problem, there exist two new collinear points L_{n1}, L_{n2} due to the potential from the circular cluster of material points. Numerical investigations reveal that with the increase in the mass of the circular cluster of material points, L_{n2} comes nearer to the more massive primary, while L_{n1} moves away from it. Owing to oblateness of the bodies, L_{n1} comes nearer to the more massive primary, while L_{n2} moves towards the less massive primary. The collinear equilibrium points remain unstable, while the triangular points are stable for $0 < \mu < \mu_c$ and unstable for $\mu_c \leq \mu \leq \frac{1}{2}$, where μ_c is the critical mass ratio influenced by oblateness of the primaries and the potential from the circular cluster of material points. The oblateness and the circular cluster of material points have destabilizing tendency.

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Keywords Restricted three-body problem · Oblateness effect · Circular cluster of material points

1 Introduction

One of the most famous problems in celestial mechanics is the well known three-body problem. The three-body problem refers to three bodies which move under their mutual gravitational attraction. Newton himself, after he explained the Kepler's laws, turned his attention into the three-body problem (Sun-Earth-Moon), but he faced some problems that remained unsolved. Almost 100 years later, by 1772, Euler proposed the restricted three-body problem (Szebehely 1967). It was the first time when the restricted three-body problem was stated. This formulation is a limit case of the three-body problem. Namely, two bodies, which are called primaries, are moving in circular orbits around their center of mass under the influence of their mutual gravitational attraction in a Euclidean space (two-body problem), while a third body, with a negligible mass, is attracted by the previous two, but not influencing their motion. The approximate circular motion of the planets around the sun and the small masses of asteroids and satellites of planets compared to the planets' masses, originally suggested the formulation of the circular restricted three-body problem (CR3BP).

The bodies in the CR3BP are strictly spherical in shape, but in actual situations, we find that some of the natural and artificial bodies moving in the space are not spherical rather they are oblate bodies. For examples, Saturn, Jupiter, Regulus and Peanut binary stars are sufficiently oblate. The minor planets and meteoroids have irregular shapes. In these cases, on account of small dimensions of the bodies in comparison with their distances, they are considered to be as

Published online: 26 October 2013